Chapter 2

Economic Theoretical Foundation of the Study

In order to profoundly study the mechanism of the existence of agricultural production and business risk, and explore to measure the risk management, it is necessary to make absorption on the elaboration of the basic economic theory that are relating to the risk analysis. Therefore, the agricultural production economics, the risk utility theory, the insurance economics, the information economics, the future and option theory etc constitute the economic foundation in the uncertainty analysis of agricultural production and business risk.

2.1 Systematic Study of Agricultural Production Economic Problem

The activities of agricultural production and business have covered the entire process in the agricultural products production and the exchange, affected each links of the production, the exchange, the consumption and the distribution in agriculture, therefore, the change in each and the existed uncertainty factors, could directly or indirectly affect the activities on agricultural production and business to bring about the risks. This chapter first implements a systematic analysis on the problems of the risk economics in the process of the agricultural production, the consumption and the exchange.

2.1.1 The Analysis Based on Agricultural Production Function

Generally, the production function which is established in an input-output relation in agriculture can be expressed as Y=f (L, K, N), among Y is the quantity of the systematic deliver, L, K, N are respectively the labor, the capital, the land and other natural resources as input factors which are given in the production process. f is a production function of the input and the output. Suppose that the agricultural net-revenue is indicated with (π), then,

$$\pi = R - C = R(Y, P) - C(w_L, w_K, w_N, Y)$$

Where, the revenue R is the function of the output Y and its price P, C is a cost function of the input-factor prices (w) and the output Y in quantity.

First, the agricultural production function uncertainty cause due to input-output portfolio in the production, since various uncertainty factors like nutrition, sunlight, temperature, moisture and so on generally exists in each stage of the reproduction processes for animals and the plants. Next, the agricultural output quantity and quality has the uncertainty, because of the dynamic change of the ecological characteristic and the organizational type of the geographical natural factors in the natural environment, which are extremely easy to injure the animals breeding and the plants growth, when the environmental factors cannot meet its physiological needs, we call they have been gotten certain kind of the disasters. The consequence of those disasters can frequently reduce the output of current agriculture or the scale of next quarter's agricultural production thus can bring the agricultural production and business risk. Therefore, when we inspect the problems of the economic risk in agricultural production, we must consider two kinds of the major factors affects, which decides the agricultural product quantity and quality. One aspect, the input-factors used for agricultural production has certain quantity and quality, the other aspect relied on the production of inner condition and the characteristic of the agricultural ecological environment.

Furthermore, in the market economy, because the cost is a function of the output quantity and the input-factor's price C(Y, w), and the revenue is the function of the output and its price R(Y, P). The agricultural production cost and the revenue include the market business risks. Such as, the stability of agricultural net revenue is either decided by the technologies used for the agricultural production process and its natural resource endowments, or by the prices stability on both markets (the agricultural product and the input-factors), which finally formed market risk. The former refers to the production risk, and the latter is named as the market risk.

The market risks contain more uncertainty factors filled up complexity. First, the purchasing cost and the input-factors price in the production process have the uncertainty, production and the circulation as agricultural products processing, the storage and the transport, the sales and so on also have the uncertainty. When these links breakdown or cannot work as normal, the undulation in the agricultural production cost does not enable the producer to obtain the anticipated revenue. Therefore the sudden or the random change in the production management, the input-factors market and the inventory's channel can frequently bring the loss to the agricultural production. Second, originally the difficulty in the market's anticipation and in the decision-making, the instability of the agricultural product price in the market often brings the uncertainty of the output quantity and the revenue.

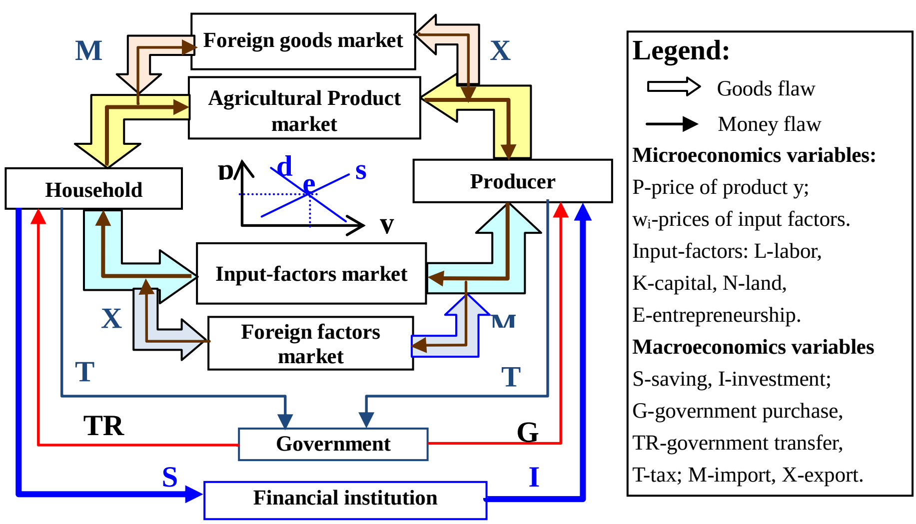

Figure 2.1 *Main Economic Variables of Affecting Agro Production and Their Relationships.*

Figure 2.1 illustrates a frame of the product market interfered or influenced by those relationships of main sections and relevant variables. The supply and demand in agricultural products market suffer from the nature and the society dual factors, especially influenced by the changes in the trade conditions in the domestic and international market, varies followed with national economic policy, financial situation, etc. It describes four sections of the micro-macroeconomic loop mode in national economy, which contains market exchange goods flows and monetary flows in an open economy, thus it can be used as the basis for us carrying on the system economic analysis from supply and demand sides. Any change on personal economic rationality, policy factors, natural selection, and domestic and international market conditions will affect the stability of equilibrium, to make the decision-making have to face many uncertainties. Thus, the product's equilibrium price frequently produces the random periodic undulation. For example, the famous economist Jan Tinbergen (1903-1994) is the earliest elaborated periodic rule of the fluctuation in the price-the spider web theory, which is often named as three types of spider web patterns [24].

2.1.2 The Analysis on Consumption Market of Agricultural Products

Agricultural production and business risk also possibly comes from the undulation in the demand in the consumption market of the agricultural product and the market of the production factors. Suppose that the consumption quantity (Y_D) of certain kinds of agricultural product is the function of its agricultural product's price and the utility, that is $Y_D = D\ (P, U)$, which is called the Hicksian demand function or the compensated demand function. In the formula, U is the consumer utility which is provided by the agricultural product measured by satisfaction degree that the consumer obtains from the consumption of certain amounts of the agricultural products. However, generally the utility function may not be observed directly, thus the Hicksian demand function may also not be observed directly, it is only indicated, the demand function which may maintain the consumer's utility at certain level through changing the price and the income. In order to explain the relations between the utility and the information of the product's quality, once in 1991 Huishung Chang and H. W. Kinnucan proposed a utility function U=U {Y [Q (I)]} [25], that the utility is a composite function of the commodity quality information Q (I). If the quality of the agricultural product is not steady, then it can affect the agricultural product's utility that brings to the consumers and creates the undulation of demand and the price in the consumption market. Obviously, improving the quality of the agricultural product will become one of the advantageous measures in guarding against the market risks.

About the consumer's behavior theory, the new classical school has done a lot of researches. Generally, the utility of the agricultural product may not only be directly expressed by the function of the consumption's quantity of the agricultural product, namely $U=U(Y_D)$, but also may be indirectly indicated for the function of the agricultural product's price and the consumers' income (money), namely u=u (P, M). Both are actually equal in the market analysis, namely u (P, M)=U as one of the important identical equations in the utility analysis, it expresses that with the income M a consumer can obtain the maximum utility. Considering $P>0$, $M>0$, from it derived Roy's identity.

Y_D (P, M) is called the Marshallian demand function. The formula indicates that demand is an ordinary market demand function where price and income can be observed directly (or the direct demand function). Therefore, whether a product's market has the stable demand, should directly inspect the change of the market's price and the consumer's income. In the agricultural products market, the demand for grain generally lacks of the elasticity on market price. So that, government generally takes the supportive price policy to stabilize or increase the income of the farmers, which is one of the important measures to averse the agricultural risk and reduce the price large undulation after the grain harvest or when the oversupply of agricultural products.

2.1.3 Analysis of Input Factors for Agricultural Production

The agricultural production regards the input-factors as not only one kind of derivation (or indirect) demand, but also a kind of union demand. The stability of the production factors market is also related to the stability of the products production, that may be proved the producer's demand for production factors directly depends on input prices and the quantity of the product, namely X=X (w, Y), here let's denote X={L, N, K, E} as a quantitative set of input-factors, and is indirectly decided by the product price and the input-factors prices, that can be expressed as X=X (P, w). Regarding the production factors have indirect changes of input demand, in which some kinds of the input factors may have the suitable substitutes, so that the uncertainty existed in their substitutes market may also be one of the reason forming agricultural production and business risks.

Since China accessed to WTO, the domestic agricultural market has further converged with the international market, means Chinese economy participated into the world economics great circulation. In the integration pattern of the world economy, the surplus and deficiency of the Chinese agricultural products may be adjusted through the international agricultural products' market in a certain extent. However, because of the enhancement of the international transaction cost, as well as the existence of the uncertainty in time and space of trade, the agricultural decision-making will involve a bigger range of the uncertainty problems from nature to social economic development. Therefore, that question reflected by the agricultural market involves the product market and the input-factor market makes the decision-making become more complex, thus the risk management of agricultural market will become more important. In order to analyze the above economic relations thoroughly, reveal the mechanism of the production theory existence, may through to setup the duality models, which relates to the market of the agricultural product and the input-factors, cost, revenue and profit, a comprehensive equilibrium analysis, which will be as a foundation for further studies.

2.2 The Duality Analysis of the Agricultural Production in Market System

To further theoretically analyze the close relations between the production market or the input factor market and the process of agricultural production and business, under the condition of the market economy, studying the in-depth reason of the existence of the decision-making risk in the market, seeking the mechanisms of the assurance, the aversion and the control of the risk. It is necessary to research the performance and the relation among the quantity of product, the essential factors, the cost, the output value, the profit and so on in the aspects of the supply and the demand market through the establishment of the dual model of production function. The discovery of their dual form is advantageous in the analysis of the decision-making risk in the process of the production business, which can help to take the effective aversion measures.

Production technology often restricts the profits obtained by producers. Here mainly refers to the technical constraints of production elasticity. The technical restriction is usually a production structural problem, because in the production process, technology determines the conversion of material, and market structure makes this transformation become more complicated. Although these problems have been concerned by economists many years ago, such as Harold Hotelling (1929), P. A. Samuelson (1947), Jan Tinbergen (1959), Ragnar Frisch (1965), R. Shephard (1970, 1981), Thomas J. Lutton and W. Erwin Diewert (1982), Hal R. Varian (1984, 1997), etc [26] [27]. However, many problems generally exist in reality, which need to prove their seriousness in theory and give more intuitive explanation. Especially the works of relate to the risk recognition in the production economy and the microeconomic system, duality analysis still need to implement the studies.

2.2.1 Definition of Multi-Input Agricultural Production Function

Suppose an agricultural production firm is faced a multi-input production function, such as, $X = (X_L, X_K, X_N) > 0$, where X_L, X_K and X_N separately express the input of the labor, the capital and the land as production factors. Then it forms three sub-vectors $X_L \in \{X_i|i=1,2,\ldots,k\}$, $X_K \in \{X_i|i=k+1,k+2,\ldots,s\}$, $X_N \in \{X_i|i=s+1,s+2,\ldots,m\}$, and let $X=(x_1, x_2, \ldots\ldots, x_m)$, $(i=1,2,...,k,...,s,...,m)$ to denote the m-dimensional input vector to indicate all input factors composition. Thus, we discuss those problems beginning with a homogeneous production function defined by the Hal R. Varian (1984), and attempt to demonstrate these derivative issues. The general hypothesis, function f is the production function expressed by the Cobb-Douglas production technology, which the generalized C-D function for a production process can be defined as following (Hal. R. Varian, 1984) [26].

$$y = f(X) = A \cdot \prod_{i=1}^{m} x_i^{b_i} \quad (i = 1, 2, \ldots\ldots, m) \quad \text{or} \quad \log y = \log A + \sum_{i=1}^{m} b_i \cdot \log x_i. \tag{2.1}$$

Here, $y > 0$ is a single output of the production, and f is the function with twice differentiated and continuous. In the formula, A as constant means total technical coefficient, b_i also express the coefficient of the production technology, which especially indicates the elasticity of production with related to the input factor of x_i, here $b_i=(\partial y/y)/(\partial x_i/x_i)=(\partial y/\partial x_i)/(y/x_i)$.

Therefore, $y=f(x)$ is strictly quasi-concave and positively with homogeneous of degree h, which respect to x* according to the significance of its economic definition. Here, h can be defined as scale elasticity, so that $h=\sum b_i$ for all x.

If there is a t>0, which makes all x time t, then we find $y_t=f(tx)=t^h f(x)=t^h y$, it is easy to be proved:

Let us define the scale elasticity as $h = \dfrac{\Delta y/y}{\Delta t/t}$. Use $\Delta t/t = \Delta x_i/x_i$ to make the total differentiation,

$$because \quad \Delta y = \sum_{i=1}^{m} \frac{\partial f}{\partial x_i} \cdot \Delta x_i = \sum_{i=1}^{m} b_i \cdot A \cdot \prod_{i=1}^{m} x_i^{b_i-1} \cdot \Delta x_i = \sum_{i=1}^{m} b_i A \prod_{i=1}^{m} x_i^{b_i} \cdot \frac{\Delta x_i}{x_i} = y \cdot \frac{\Delta t}{t} \cdot \sum_{i=1}^{m} b_i,$$

so we get the scale elasticity $\dfrac{\Delta y/y}{\Delta t/t} = \sum_{i=1}^{m} b_i = h$.

Y is called the homogeneous function of degree h, to X it can be tested by use t > 0, and find

$$y_t = f(tX) = A \cdot \prod_{i=1}^{m} (tx_i)^{b_i} = t^{\sum_{i=1}^{m} b_i} \cdot A \cdot \prod_{i=1}^{m} x_i^{b_i}.$$

Especially, when h=0, the Y is named homogeneous function of degree zero, if h=1, it's the homogeneous function of degree one.

2.2.2 The Profit Maximization Model

In market system, the market prices have to be decided by demand and supply. The equilibrium analysis generally discusses how to decide the price, and to determine the optimal input or output. It is well known that production and management decision pursuits economic profit. As noted above, thus after we have assigned the price P of the single product and the prices of each input factors as $w \in \{w_L, w_K, w_N\} = (w_1, w_2, \cdots; w_m) > 0$, and we may establish a profit function, $\pi = \pi(P, w, X)$. In this case, a profit function can be defined as the following. [26]

$$\pi = p \cdot y - w \cdot X = p \cdot y - \sum_{i=1}^{m} w_i \cdot x_i \tag{2.2}$$

From (2.2) easy to prove that $\pi = \pi$ (P, w, X) is zero-order degree of homogeneous function with (p, w). To all (i=1, 2... k...s...m) at a general market, it is easy to prove two famous lemmas:

$$y = \partial\pi/\partial p \text{ (Hotelling); } x_i = -\partial\pi/\partial w_i \text{ (Shephard).}$$ [28]

Therefore, if to get the Profit Maximization, we have to determine the input values of X*>0, which make that

$$\pi(P,w,X^*) > \pi(P,w,X) \text{ for all } X > 0,\ w > 0,\ P > 0.$$

Suppose that the profit function has the linear relation regarding the prices of the product and the input factors. Therefore, in certain product market to obtain Maximum π that means the first order conditions have to be deviated by the defined profit function and production function. Here, based on formula (2.2) let $\partial\pi/\partial x_i=0$ as the first order condition for optimal input x_i and optimal profit.

Thus, we obtain the first order condition by taking the partial derivatives as following

$$\partial\pi/\partial x_i = p\cdot \partial y/\partial x_i + y\cdot \partial p/\partial y \cdot \partial y/\partial x_i - w_i = 0,$$

So that we find $p\cdot \partial y/\partial x_i \cdot \left(1 + y/p \cdot \partial p/\partial y\right) - w_i = p\cdot \partial y/\partial x_i \cdot \left(1 + 1/\varepsilon_p\right) - w_i = 0.$

Where $\varepsilon_p = \dfrac{\partial y/y}{\partial p/p}$ is usually defined as the price elasticity of demand.

In general product market, $\varepsilon_p<0$ is defined as the demand price elasticity of product y. Especially, in perfect competition market when the producer is a price taker that $|\varepsilon_p|\rightarrow\infty$ must be required, however, in the market of imperfect or monopoly $|\varepsilon_p|>1$ will be the condition. Therefore, formula (2.3) is easy to get as a solution of profit maximization for monopoly market.

$$\partial y/\partial x_i \cdot \left(1 + 1/\varepsilon_p\right) = w_i/p > 0 \tag{2.3}$$

I. The Optimal Input-Factors Determine in Different Market

Because of the formula is required by the general property of optimal profit, it means y is increasing in x, and (p, ω)>0, which makes $|\varepsilon_p|>1$. However, if we consider to derive factor set x^* as results for Maxπ in different markets. Using formula (2.1), due to the definition of A and b_i are constants, the marginal product as the first order derivation can be written as

$$\frac{\partial y}{\partial x_i} = \frac{\partial}{\partial x_i}\left(A \cdot \prod_{i=1}^{m} x_i \right) = b_i \frac{y}{x_i}.$$

By means of the equation (2.3) we get

$$b_i \frac{y}{x_i}\left(1+ {}^{1}\!/_{\varepsilon_p}\right) = \frac{w_i}{p}.$$

Thus, the solutions of the formula for arbitrary market can be written as following:

$$x_i^*(p, w, y) = \begin{cases} p \cdot y \cdot b_i \,/\, w_i & (2.4\text{-}1) \\ p \cdot y \cdot \left(1+1/\varepsilon_p\right) \cdot b_i \,/\, w_i & (2.4\text{-}2) \end{cases}$$

Especially, (2.4-1) refers to an optimal input x_i^* under the perfect competition market, it is derived by price supposed as a constant or $\varepsilon_p \to \infty$, as well as in taking limit method to be proved by

$$\lim_{\varepsilon_p \to \infty} \left(1 + 1/\varepsilon_p\right) = 1 .$$

However, if the case is an imperfect market, we can determine the equilibrium input x_i^* by (2.4-2). Because of $(1+1/\varepsilon_p)<1$ in monopoly market, implies that $|\varepsilon_p|>1$. If a maximum profit relies on the optimal input vector x^*, it's easy for us to use formula (2.4) to find $\pi(P, w, X^*) > \pi(P, w, X)$ for all X > 0, w > 0, P > 0 over the related market.

II. Maximum Profit Function and Its Properties

The optimal profit model through above definition can be derived as following:

$$\pi^* = \pi(p, w, x^*) = p \cdot y\text{-}w \cdot x^* = p \cdot y\text{-}\sum_{i=1}^{m} w_i \cdot x_i^*$$
$$= \begin{cases} p \cdot y \cdot (1-h) & (2.5\text{-}1) \\ p \cdot y \cdot [1-(1+1/\varepsilon_p) \cdot h] & (2.5\text{-}2) \end{cases}$$

Certain information and knowledge inferred by formula (2.5), a direct knowledge is about the revenue p·y, the key signal that is about elasticity. Which are more important economic signals include the scale elasticity of production $h=\sum b_i$ and the price elasticity of demand ε_p. Elasticity shows the stability of production process or the sensitivity to input factors and market prices. Those will affect the quantity of profit effectively, when we analyze on the production function, particularly the values of h and ε_p need to be measured and well controlled. It is necessary to identify those parameters, which characterized the economic system risks and uncertainties existence.

Currently most of markets structures belong to imperfect competition, monopoly elements are ubiquitous. The influence of monopoly in formula (2.5-2) is indicated by price elasticity of demand ε_p. Therefore, $|\varepsilon_p|>1$ is a basic requirement that postulated to be reached in market of monopoly production. Similarly, perfect competition market look at formula (2.5-1) can be existed only when $\varepsilon_p \to \infty$ or $(1+1/\varepsilon_p) = 1$. To compare those two kinds of markets for $\pi^* \geq 0$, which requires $h \leq 1$, but this only relates to the process of production, however, the sensitivity variation of market demand to product's price depend on $(1+1/\varepsilon_p) \cdot h$. So that, $0 < (1+1/\varepsilon_p) < 1$ is the normal case of imperfect competition market, considering $h \leq 1$, thus the optimal profit in perfect competition market general no more than it's in imperfect competition market, because of $p \cdot y \cdot (1-h) \leq p \cdot y \cdot (1-(1+1/\varepsilon_p) \cdot h)$.

III. The General Properties of Profit Function

Typically, during above analysis the profit $\pi^*(p, w)$ is used to be endogenous variable, which will be determined by market prices system (p, w), but the price and technology are exogenous variables. Under this case, the factors of choice are also endogenous variables[4]. Therefore, the properties of profit function $\pi^*(p, w)$ are easy to be proved herewith.

(i) Profit function is non-decreasing in p for output, non-increasing in w for inputs. That means $p' \geq p$ and $w' \leq \omega$, then $\pi^*(p',w') \geq \pi^*(p,w)$.

(ii) $\pi^*(p, w)$ is homogeneous of degree one in prices (p, w). $\pi^*(tp, tw) = t\pi^*(p, w)$ for $t > 0$.

(iii) Profit function is convex in (p, ω).

Let $p'' = t \cdot p + (1\text{-}t) \cdot p'$, for $t \in [0,1]$, then $\pi^*(p'') \leq t \cdot \pi^*(p) + (1\text{-}t) \cdot \pi^*(p')$.

(iv) Profit function $\pi^*(p, w)$ is continuous in (p, w), at least for $p>0$, $w \gg 0$, and π^* is convergent conditionally with $h<1/2 \ll |\varepsilon|$ (see formula 2.6).

(v) In addition, it's easy to demonstrate $\partial x_j / \partial x_i = w_i / w_j$ by formula (2.3).

2.2.3 Supply and Demand Functions

I. The Supply Function

If we take the results in formula (2.4) bring into (2.1), a supply function of producer for optimal profit can be obtained. This result shows in formula (2.6), which supply function y (p, w) in different markets the issues will be also different. Thus

4 Hal R. Varian. "Microeconomic Analysis" (3ed). New York, W.W. Norton & Co., Inc. 1992: p41-43, p203. That means the exogenous and endogenous may be changed in economics analysis depends on the different models, generally the exogenous decided the endogenous. Such as, in nature sunspot always influenced and decided the earth agro production, so environmental factors used to be exogenous variables.

$$y^* = f(x^*) = A \cdot \prod_{i=1}^{m} (x_i^*)^{b_i} \qquad \text{for all } (i = 1, 2, \ldots\ldots, m)$$

$$= y^*(p, w) = \begin{cases} \left[A \cdot p^h \cdot \prod_{i=1}^{m} \left(b_i / w_i \right)^{b_i} \right]^{1/(1-h)} & (2.6\text{-}1) \\ \left[A \cdot p^h \cdot \left(1 + 1/\varepsilon_p \right)^h \cdot \prod_{i=1}^{m} \left(b_i / w_i \right)^{b_i} \right]^{1/(1-h)} & (2.6\text{-}2) \end{cases}$$

There are many properties in formula (2.6), where (2.6-1) is perfect competition market, and (2.6-2) is an imperfect competition market. Such supply function as the optimal output of y^* is monotonically increasing with both price p and $|\varepsilon_p|$, the price elasticity of supply function of the output that indicated by 1> h/(1-h)>0 should be limited to small one, thus if h/ (1-h)<1 requires h<1/2. Moreover, in monopoly market both h<1/2 and 1> $(1+1/\varepsilon_p)$> 0, as well as $|\varepsilon_p|$>1 become necessary.

Considering profit formula (2.5) and using supply function (2.6), the optimal profit function can be improved at following

$$\pi^*(p, w) = \begin{cases} \left[A \cdot p \cdot \prod_{i=1}^{m} \left(b_i / w_i \right)^{b_i} \right]^{1/(1-h)} \cdot (1-h) & (2.7\text{-}1) \\ \left[A \cdot p \cdot \left(1 + 1/\varepsilon_p \right)^h \cdot \prod_{i=1}^{m} \left(b_i / w_i \right)^{b_i} \right]^{1/(1-h)} \cdot \left[1 - \left(1 + 1/\varepsilon_p \right) \cdot h \right] & (2.7\text{-}2) \end{cases}$$

As can be seen from the formula (2.7), for profit may depend on technology A, market price (p, w) and elasticity (h, b_i), rather than the quantity of inputs and outputs to determine. So we can name them as indirect profit functions. All manifested properties above profit function by use the formula (2.7) can be testified clearly.

II. The Demand Function

Furthermore, we can derivative demand function of input-factors by (2.4) and (2.6), which are the optimal results. We obtain the formula (2.8) of demand function x (p, w) as final forms.

$$x_i^*(p, w, y) = \begin{cases} p \cdot y^* \cdot b_i / w_i = x^*(p, w) = \left[A \cdot p \cdot \left({}^{b_i}\!/\!_{w_i} \right)^{1-h} \cdot \prod_{i=1}^{m} \left({}^{b_i}\!/\!_{w_i} \right)^{b_i} \right]^{1/(1-h)} & (2.8\text{-}1) \\ p \cdot y^* \cdot \left(1 + 1/\varepsilon_p\right) \cdot b_i / w_i = x^*(p, w) = \left[A \cdot p \cdot \left(1 + 1/\varepsilon_p\right)^h \cdot \left({}^{b_i}\!/\!_{w_i} \right)^{1-h} \cdot \prod_{i=1}^{m} \left({}^{b_i}\!/\!_{w_i} \right)^{b_i} \right]^{1/(1-h)} & (2.8\text{-}2) \end{cases}$$

As a well known property for demand function has homogeneous of degree 0 in (p, ω), which is easy to demonstrate in the final forms of (2.8). There is x*=x (t ·p, t ·w) = x (p, w), for a given t>0, both can be testified. Therefore, the monopoly factors of imperfect competition market, through demand price elasticity and production elasticity cause impacts to profit. Even thought, monopoly may increase profit of producer than profit in perfect competition market, but it constraint the quantity of product and limit the demand growth. Those may also prove that market monopoly make production lose efficiency, or lead to the market failure.

2.2.4 The Cost Minimization Model

As a duality study of profit maximum is the cost minimization, the model generally use C-D production function (2.1) to express y, a cost minimum model can be shown at following [26]:

$$C\,(w, y) = \mathrm{Min}\,(w \cdot X)$$

$$\text{ST.} \qquad X \in V(y).$$

Regarding all w, y, if the cost function C (w, y) <w ·X, then the model becomes:

$$\mathrm{Min}\; C(w, y) = w \cdot X$$

$$\text{ST. } y<f(X),\text{and } X,w>0. \tag{2.9}$$

We may solve the question (2.9) by use the Lagrange multiplier method, if there is a λ>0, let

$$\text{Min } \zeta(w, y) = w \cdot X - \lambda\, [y-f(X)].$$

Therefore, through the first order partial derivation we have:

$$\frac{\partial \zeta}{\partial x_i} = w_i - \lambda \frac{\partial f(X)}{\partial x_i} = 0, \ (i=1,2,\ldots,k,\ldots,s,\ldots,m), \tag{2.10}$$

$$\frac{\partial \zeta}{\partial \lambda} = Y - f(X) = 0. \tag{2.11}$$

From (2.10) we get,

$$\frac{w_1}{\partial f/\partial x_1} = \frac{w_2}{\partial f/\partial x_2} = \ldots\ldots = \frac{w_m}{\partial f/\partial x_m} = \lambda, \qquad y = f(X).$$

Similarly, we use $\partial y/\partial x_i = b_i \cdot y/x_i$ that can be derived from (2.1) to get

$$\frac{w_1 x_1}{b_1} = \frac{w_2 x_2}{b_2} = \ldots\ldots = \frac{w_m x_m}{b_m} \tag{2.12}$$

Therefore, regarding all (i=1,2,…,m), the producer's direct demand function and the cost function of the production factor are given as follows [29]

$$x_i(w, y) = \left(\frac{b_i}{w_i}\right) \cdot \left[A \cdot \prod_{i=1}^{m} \left(\frac{b_i}{w_i}\right)^{b_i}\right]^{-1/h} \cdot y^{1/h}, \tag{2.13}$$

$$C(w, y) = h \cdot \left[A \cdot \prod_{i=1}^{m} \left(\frac{b_i}{w_i}\right)^{b_i}\right]^{-1/h} \cdot y^{1/h}. \tag{2.14}$$

May prove that, regarding formula (2.13) and (2.14), the demand function X (w, y) is a homogeneous of degree 0 in w, but the cost function C (w, y) is homogeneous of degree one in w. Both of them are the increasing functions of y.

In brief, once confirm to the condition of the previous hypothesis, what decision-making the agricultural production business rests on should be the functional relations and the natures in the former models, it is extremely essential to the risk recognition and the risk analysis. Through the analysis in the above model, we may obtain several important conclusions as follows [29] [30].

First, according to formula (2.7), the profit function $\pi^*(p, w)$ is homogeneous function of degree one in (p, w), is the increasing function of the products price (the quasi-convex) and the decreasing function of the input-factor price. A necessary condition of the profit $\pi^*(p, w) > 0$ is $h < 1$, therefore in order to obtain the stable revenue and supper profit, the production function must have the decreasing return to scale. According to formula (2.6), the indirect production function $y^*(p, w)$ also names the supply function, which is reduced regarding the input factor's price (w), but it's increased regarding to the product's price p. When the supply is lack of the price elasticity has $h / (1-h) < 1$, then there is a big limit to request of $h < 1/2$ in the elasticity of production scale.

Second, based on the significance of the elasticity, production and consumption, supply and demand by market closely interacted linked. Especially at monopoly market, regarding (2.7-2) we find the links among the elasticity.

If has $[1-(1+1/\varepsilon_p)\cdot h]>0,$ when $0<(1+1/\varepsilon_p)<1,$ h>0, then $h>\varepsilon_p/(\varepsilon_p+1), \forall\varepsilon_p\in(-\infty,-1)$.

That shows the output has increasing return to scale, the demand price is elastic, but there isn't a single solution, means monopoly market makes input factors lose efficient utilization.

Third, the inputs demand function can be divided as the direct and the indirect demand functions. According to formula (2.8), the indirect factors demand function $x_i(p, w, y)$ is homogeneous of degree zero in price (P, w). But the demand elasticity of the product's price is 1/(1-h), if want the profit $\pi^* > 0$, then $h < 1$, which implies $1/(1-h)>1$ namely the input demand is ample in elasticity to the price of the product. This explains that the small change of the products price can cause the momentous change of the input demand and both are changed in the same direction.

Fourth, about direct factors demand function without consider the monopoly of input market, in formula (2.13) x_i (w, y) is a homogeneous of degree 0 in w, increasing in y. Where the price of the product is not considered in the formula, but the elasticity of the output y is 1/h, as the limit of $0 < h < 1$, the direct demand of the input-factor is lack of elasticity regarding the output of the product. Formula (2.14) implies cost function is quasi-concave with homogeneous of degree one in w, and increasing in (w, y). So that, x_i (w, y) as conditional input-factors demand is determined only by cost and output.

Fifth, through the analysis on the duality, when portrays the process of one special production with the homogeneous function, the above inferential reasoning conclusions must be considered. According to some scholars, like J. Michael Price (1994), who once suggested guaranteeing the profit maximization, the input demand function of in-elasticity is inconsistent with these limits results. Therefore, if certain kind of the signs indicates the input demand is inelastic regarding the change of the output price, thus it is not convenient to use the homogeneous production function to simulate the production's process [30]. Many inputs in the agricultural production belong to this situation regarding the changes of the products price; however, this cannot hinder the application of the entire model in the practical research.

Finally, the above analysis in the model is taking the Cobb-Douglas homogeneous production function as the example. There is a kind of the theory significance in the process of the analysis, when we make the risk decision, we must consider every economical variable and the mutual relations between them, and how they do finally affect the risk income or the risk lose in the production and the business. Especially when the production functional relation itself frequently has the uncertainty and the massive existence of the relations of discrete or non-linear, which may add the complexity to our decision-making? Although sometimes the input demand is inelastic regarding the changes of the output price, the above analysis process is still helpful for us to discover the existence mechanism of the risk elements.

2.3 The Risk Utility Theory

The utility is originally refers to the people's satisfaction degree that obtains from the economic goods, conceptually the utility includes the factors which influence the risk decision's aspects of the psychology, the culture and so on. In the research of the uncertainty economic theory, the utility theory attempts to use one sole standard such as the utility unit or the currency unit to carry on the analysis and to measure the risk. The modern economics as one of the tools for the empirical analysis of the risk and it has provided the explicit and confirmed the results on the question of the individual understands the risk. To measure of the risk, the policy-makers' information are generally asymmetrical or incomplete, in addition to the different individual's experience, the institution and the preference, the individual's manner to treat the identical risk may be different, thus they possibly take the different methods. Therefore, the risk preference belongs to the individual's subjective factor and the risk preference has decided the countermeasures of business main body, when it faces the risk that he rather to withstand in order to obtain certain revenue, as well as the cost that he is willing to pay to averse the risk.

People have already accumulated many original concepts of risks phenomenon in the long-term economic life. For example, the bigger the risk is, the more it worth to averse. The higher the harm's probability is, the bigger the risk is, the more precious of the property, the more cherish are worth and so on. Therefore, based on those none debated facts to establish the inference rule of the risk economic problem, and derived a set of the scientific theoretical system that is constituted by the theory or the law according to the logic, this is the axiomatic method.

2.3.1 The Risk Measure Axiom

The risk measure has two indispensable groups of the existence axiom. The first group is the comparison rule of the risk measure, is the dual relation who is established in the general set that is defined on the risk distribution's collection. These dual relations have many intrinsic attributes, like reflexivity, non-reflexivity, symmetry, asymmetry, anti-symmetry, transitivity, negative transitivity, completeness, weak completeness. According to the concrete study objects, we may choose several kinds of essential dual relational attributes and integrate them to the axiom-system of the risk measure. For example, one rational actor, under the condition that the information is relatively complete, always can aim at the known risk and the size of the harm and the loss, and discharge their orders according to the risk's degree. This ordered axiom generally includes the reflexivity, transitivity and completeness. At the same time, as the behavior's feelings regarding the risk may have the corresponding changes along with his loss harmfulness gradually. Therefore, it is needed to introduce the second group of the axiom, namely the Archimedes axiom. Two groups of the axiom constitute the existence axiom of the risk comparison and the risk measure. After the inference we may also get other axioms or theorem such as the union axiom, and the algebra nature and so on.

In the ordinal utility theory, U(x) is a consumer's utility for certain kind of goods x, if he believes, the utility of y is not bigger than the utility of z, then the expression is $U(y) \le U(z)$, or $y \le z$. For the risk question, if "the risk of u is no bigger than the risk of v", then we may express this kind of weak preference in using mark "$\precsim$" to judge ordering, and write it as $u \precsim v$. Therefore, the weak ordering axioms to the risk measure have following three [31]:

The weak ordering axiom If $\aleph$ is the collection of the risk distribution, then define the dual relation $\precsim$ of "the risk is not bigger than" in $\aleph$, which have to meet the following natures.

(i) Reflexivity: For all $u \in \aleph$, $u \precsim u$.

(ii) Transitivity: For all u,v and $w \in \aleph$, if $u \precsim v$, $v \precsim w$, then $u \precsim w$.

(iii) Completeness: For all u, $v \in \aleph$, either $u \precsim v$ or $u \succsim v$ or both.

The assumption (i) is trivial, and (ii) is necessary for discussion risk preference maximization, if without the transitivity we are impossible to make the optimal choice. (iii) means any two bundles can be compared, which has established the suppositions for the dual comparison. Moreover, except for the existence of the relation of the weak ordering, we also frequently experienced the relation of the strictly ordering in reality, for example "the risk u is strictly smaller than the risk v", and then the mark is $u \prec v$. Similarly, we also may define that the risk u and the risk v are indifference, namely when there are the dual relations of $u \precsim v$ and $u \succsim v$, the mark $u \sim v$ or called equivalent relation [26].

(iv) Continuity: For all u, $v \in \aleph$, there are $\{u \succsim v\}$ and $\{u \precsim v\}$ are closed sets. Then $\{u \succ v\}$ and $\{u \prec v\}$ are open sets.

The assumption is necessary to rule out certain discrete behaviors. The continuity as an important consequence, if $\aleph$ is a set of the risk distribution, and the relation of weak ordering $\precsim$ has also been defined in $\aleph$, then to arbitrary $u,v,w \in \aleph$, the subset $\{\alpha \in [0,1] | (1-\alpha)u+\alpha w \precsim v\}$ and $\{\alpha \in [0,1] | (1-\alpha)u+\alpha w \succsim v\}$ in the real number interval [0,1] must be a closed set. If defined the strictly ordering relation $\prec$ in $\aleph$, then to arbitrary $u,v,w \in \aleph$, the subset $\{\alpha \in (0,1)|(1-\alpha)u+\alpha w \prec v\}$ and $\{\alpha \in [0,1]|(1-\alpha) u+\alpha w \succ v\}$ which is on the real number interval (0,1) must be an open set.

The establishment of risk ordering axiom, has laid a foundation for the positive study and the quantitative comparison, and has a mathematical significance strictly to the functions natures in measure risk. In actual research, about other risk measurements there are usually the following two assumptions.

(v) Strict monotony: For $u,v \in \aleph$, if $u \precsim v$ and $u \neq v$, then $u \prec v$; Vice versa, then $u \succ v$.

(vi) Strict convexity: Given $u \neq v, w \in \aleph$, if $u \succsim w$ and $v \succsim w$, then for all $\alpha \in (0,1)$ has $\alpha u+ (1-\alpha) v \succ w$. Then others are vice versa.

Therefore, for all $u,v,w \in \aleph$, may always have $\alpha, \beta \in [0,1]$ and $\gamma = \alpha \beta$, which cause the existence of the following two algebra relations.

$$\alpha u+ (1-\alpha) v= (1-\alpha)v+\alpha u$$

$$\alpha \ (\beta u+ (1-\beta) v) +(1-\alpha)v=\gamma u+ (1-\gamma) v$$

The application of the former axiom and the assumptions, we may conduct further research on the utility theory of the risk aversion.

2.3.2 The Utility of the Risk Aversion

I. Risk and Expected Utility

The risk is not only an absolute concept, but also a relative one. The ability of individual withstanding of risk has his own size, but that doesn't mean he certainly is a risk lover. Based on the assumption of personal rationality in new classical school, anybody can be seen himself as a risk aversion [2]. Theory of risk utility has provided a valuable logical frame for decision-making under the uncertainty condition. The so-called behaviors of the risk aversion refer to expected utility of the business activities will be strictly lower than the utility of expected value.

May use Jensen's inequality Ef(x) <f (Ex) to prove this issue [9]. "Therefore, the concavity of the expected utility function and the risk aversion are synonymy" (Varian, 1992) [26]. Suppose f (x) for the probability density that defined in x, as a consequence related to certain kind of the business activity, then the expected utility function is:

$$EU = \int u(x)\, f(x)\, dx\,.$$

Suppose a business is risk aversion, then EU>E[f(x)], thus he set up his utility function by two rules: The wealth quantity increasing will make him more satisfying. And his marginal utility is diminishing when the wealth is increasing. The curve of the expected utility function can be illustrated in Figure 2.2.

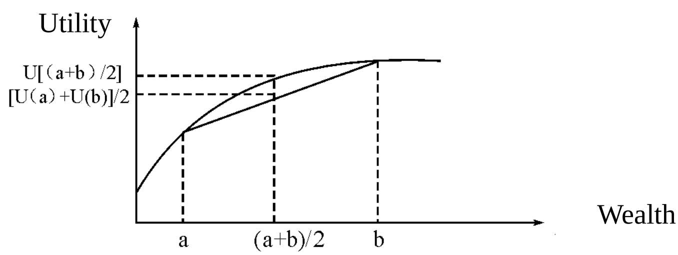

Figure 2.2 *Expected Utility.*

Intuitively, the risk measure can be illustrated by the curves of the expected utility. The more concave of the curve is, the stronger tendency of the operator's risk aversion is, and vice versa. Usually we use wealth utility curve to represent the different risk preference, the slope of a curve is the marginal utility (MU) for every increase one-unit wealth, therefore, from the shape we can judge the attitude of the decision-maker to risk.

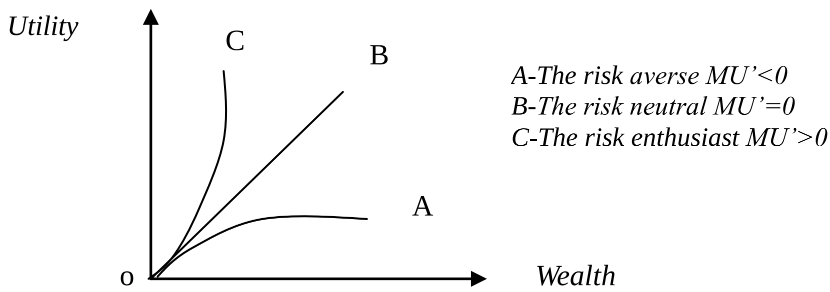

Figure 2.3 *Wealth Utility Curves of Different Risk Preferences.*

The Figure 2.3 illustrates the three wealth utility curves that respectively represent three kinds of different risk preference. Like what shows in chart A, the risk aversion is a person whose marginal utility is in diminishing, he is willing to give up the wealth when the loss has not occurred, but once when the lose occur, he may obtain the compensation of the wealth. But the manner of a risk enthusiast is the opposite, such as what shows in chart C, its marginal utility is increasing. And a risk neutral, what shows in chart B, its marginal utility is in constant.

The curves in Figure 2.3 also can be expressed approximately by the mathematical formula, let $U \in [0,1]$ to express the value of utility, and $x \in [a, b]$ as the value of wealth (for example the currency value). Whatever, the attitudes of people regarding the risk, all may have the expression as following [32].

$$U(x)=\begin{cases} 1-\left(\dfrac{x-b}{a-b}\right)^{\lambda}, & \lambda>1, \quad \text{Risk Averse;} \\ \dfrac{x-b}{a-b}, & \lambda=1, \quad \text{Risk Neutral;} \\ \left(\dfrac{x-a}{b-a}\right)^{\lambda}, & \lambda>1, \quad \text{Risk Enthusiast.} \end{cases}$$

The risk utility theory relates to the question that a person's understanding of the risk, what has been applied more common in the decision-making is the subjective probability. The overseas psychology theories that relate to the risk recognition believe that the objective risk is a kind of fallacy. Advocated that the risk itself is subjective, it is influenced by the society, the politics, the culture, the psychology and as well as other series of factors. So-called "the recognition" is taking the existence of the objective risk as the premise, and argues that the risk is independent from people's explanation and the culture, which waiting for people to survey. That is the risk in the figures, which infer from the supposition, the model and as well as the formula that established by people themselves. That means, risk recognition has the dynamic property, when personal to process risk will have certain limitation (Harold D. Skipper, Jr. 1998) [2]. We believe that deal with risk not only has an objective side, but also has a subjective side, since the risk should be a kind of deviation that people's subjective reflects on the objectiveness. In the real life the people's manners to the risk can have the change along with the enhancement of their income, because the difference among the treatment of different producer in the risk utility. When balancing the risk and the profit, their behaviors are possibly different or may change, in a great extent this is decided by the risk decision-maker's subjective wish and the change of their preference. The risk aversions are willing to reduce the risk by purchasing expensive tools or to transfer scatter the risk to others, but the risk amateurs are the opposite to deduct.

II. Arrow-Pratt Risk Aversion Coefficient

The risk aversion measure generally uses the second order derivative of the expected utility function. Intuitively the more concave the expected utilities function, the more risk averse of the decision-maker. In order to have its normalized definition, we get a reasonable measure known as the "Arrow-Pratt measures of (absolute) risk aversion", which is the second derivative by dividing by the first derivative, or take derivative by the logarithmic function of the first derivative of the expected utility [26] [33].

$$\gamma(x) = -{u''(x)}\big/{u'(x)} = -{d\ln(u'(x))}\big/{dx}.$$

Assume this utility function is U(x) = -exp (-ρx), then the Arrow-Pratt risk aversion coefficient is:

$$\gamma(x) = -\frac{-\rho^2\exp(-\rho x)}{\rho\exp(-\rho x)} = \rho$$

Therefore, the above three kinds of situations can also be expressed by the coefficient of the absolute risk aversion.

$$\gamma(\mathrm{x}) = \begin{cases} >0 & \rho>0 \ \text{ as type of the risk aversion;} \\ =0 & \rho=0 \ \text{ as type of the risk neutral;} \\ <0 & \rho<0 \ \text{ as type of the risk enthusiast.} \end{cases}$$

In practical application also has the coefficient of the Arrow-Pratt relative risk aversion, except for the coefficient of the absolute risk aversion. The coefficient of the relative risk aversion can use the formula $\gamma_R(x) = -x\cdot {u''(x)}\big/{u'(x)}$ to express.

The significance of the risk aversion coefficient is explained by the marginal utility, it expressed is the marginal utility in percentage of varies under any income level, namely “each unit income change brings the marginal utility”. Therefore, this proposes a question between the absolute risk aversion and the income level, whether the part risk aversion can represent the change of the overall scale? For example, in agriculture, the question whether the utility of each unit cultivated area can represent as the entire agriculture. This is the problem of scale transformation of using the Arrow-Pratt risk aversion coefficient; regarding this we have the following two theorems:

The theorem 1: Let $\gamma(x) = {u''(x)}\big/{u'(x)}$ defined as a scale transformation of X, thus there is a X = CY, and C is a constant, which will bring us:

$$C\gamma(x) = \gamma(y)$$

That easy to prove: to substitute X=CY into the formula of the utility function in U (X) = U (CY), then by the first and the second order derivative in Y on the utility function, we have,

$$\begin{aligned}
&\frac{\mathrm{dU}}{\mathrm{dy}} = \frac{\mathrm{dU}}{\mathrm{dx}}\frac{\mathrm{dx}}{\mathrm{dy}} = \mathrm{CU}'(\mathrm{x});\\
&\frac{\mathrm{d}^2\mathrm{U}}{\mathrm{dy}^2} = \frac{\mathrm{d}}{\mathrm{dy}}\left(\frac{\mathrm{dU}}{\mathrm{dy}}\right) = \mathrm{C}^2\mathrm{U}''(\mathrm{x});\\
&\gamma(\mathrm{y}) = {\mathrm{d}^2\mathrm{U}/\mathrm{dy}^2}\big/{\mathrm{dU}/\mathrm{dy}} = \gamma(\mathrm{x}/\mathrm{C}) = \mathrm{C}\gamma(\mathrm{x}).
\end{aligned}$$

Namely indicate that the production scale and the size of the risk aversion coefficient are proportional changing in same. This seems a good explanation why in the cooperation economy, follows the scale expand of cooperation will bring more risk factors, particularly for land cooperatives along its scale augment the level of risk will increase.

Theorem 2: If there is Y=X+C, C as a constant, then $\gamma(y) = \gamma(x)$.

Indicate that the coefficient of the absolute risk aversion isn't influenced by the size of the scale constant or the absolute change. That is, the risk level of a system is due to its structural varying or results from some factors changing.

2.3.3 The Risk and the Revenue

The risk and the revenue as two important factors must be considered in the risk decision process. What the risk brings are the negative utility, but what the revenue brings are the positive utility. If taking the expected utility as the basis of the decision-making, thus we can establish the following expected utility function $E(U) = f[E(R), \sigma]$, among *E(U)* expected utility, *E(R)* expected revenue, σ standard deviation. Generally the expected utility is increasing along with the increase of the expected revenue, but the expected utility is the reducing along with the increase of the standard deviation. The formula is as following $\partial f/\partial E(R)>0, but\, \partial f/\partial \sigma <0$

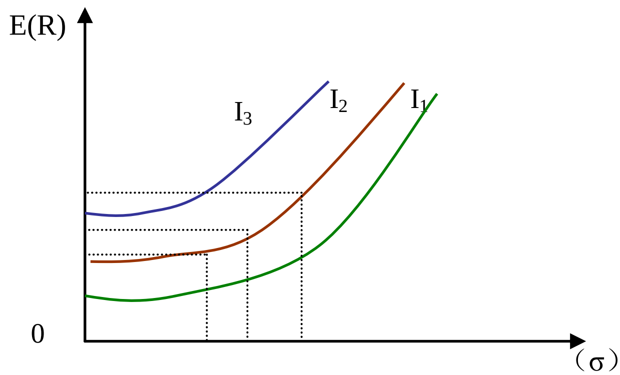

Figure 2.4 *Risk-Revenue Indifference Curve.*

When we illustrate the different composing of risk and revenue in a two-dimensional figure, we may obtain a group of indifference curves, where each curve has the single value of utility. "The Risk-Revenue Indifference Curve [3]" of the risk averse is shown in Figure 2.4. It usually has the following properties:

(i) $dE(U)=\frac{\partial f}{\partial E(R)}dE(R)+\frac{\partial f}{\partial \sigma}d\sigma =0$ explains that in an individual curve the expected utility is indifference, each point of the composition of risk and revenue has identical utility.

(ii) $\frac{dE(R)}{d\sigma}=\frac{\partial f/\partial \sigma}{\partial f/\partial E(R)}>0$ indicate that its slope is positive, thus the reason why people are willing to withstand the higher risk is because they want to obtain higher revenue.

(iii) $\frac{d^2E(R)}{d\sigma^2} > 0$ indicates that the indifference curve concave to the ordinate axis, and the expected revenue in the figure has the minimum value. Namely along with the increase of the risk, the expected revenue is increasing, the equal risk requests more revenue to compensate. When maintaining the same expected utility, the marginal substitution rate between the risk and the expected revenue is increasing.

(iv) In Figure 2.3, $I_1 < I_2 < I_3$ indicate that every two indifference curves which cannot be intersected, and the utility is growing along with the shift to the top of the left.

2.3.4 The Utility Principle of the Risk Income

The typical principle is, the operators (the consumer, the producer and so on) are always seeking for the maximum expected utility (or cash income). Supposes x_1 and x_2 are respectively the consumer goods, p_1 and p_2 are respectively the corresponding prices, Y is the cash income, U is the consumer's utility, α and β are respectively the utility elasticity of two kinds of goods, which are also called the coefficient of Cobb-Douglas (C-D) utility function. Thus, the model is expressed as the following:

$$\underset{x_1,x_2}{Max} U(x_1, x_2) = x_1^{\alpha} x_2^{\beta} \qquad st\ p_1x_1 + p_2x_2 \le Y.$$

Since the concavity of the utility function, the inequality can be replaced by the equality, then transform the above maximized equation into the Lagrange form.

$$L = x_1^{\alpha} x_2^{\beta} + \lambda(y - p_1x_1 - p_2x_2)\ .$$

Therefore, we can obtain the first order condition:

$$\partial L/\partial X_1 = \alpha\ X_1^{\alpha}\ X_2^{\beta}\ /X_1 - \lambda p_1 = 0 \qquad (2.15)$$

$$\partial L/\partial X_2 = \beta X_1^{\alpha} X_2^{\beta}/X_2 - \lambda p_2 = 0 \tag{2.16}$$

$$\partial L/\partial \lambda = y - P_1 X_1 - P_2 X_2 = 0 \tag{2.17}$$

By (2.15)/(2.16):

$$x_2 = \frac{\beta}{\alpha}\frac{p_1}{p_2}x_1 \tag{2.18}$$

Bring (2.18) substitute into (2.17), we get:

$$x_1(p_1, p_2, y) = \frac{y}{p_1}\left(\frac{\alpha}{\alpha+\beta}\right) \tag{2.19}$$

Similarly, we can get:

$$x_2(p_1, p_2, y) = \frac{y}{p_2}\left(\frac{\beta}{\alpha+\beta}\right) \tag{2.20}$$

In equation (2.19) and (2.20), x is the function for price and income, generally so-called Marshallian demand function. Substituting it into the direct utility function $U(x_1,x_2)$, we get the indirect utility function (2.21), this indicates that the optimal behavior of the decision-making directly relies on the income and the price.

$$v(p_1, p_2, y) = \frac{y^{\alpha+\beta}}{(\alpha+\beta)^{\alpha+\beta}}\left(\frac{\alpha}{p_1}\right)^{\alpha}\left(\frac{\beta}{p_2}\right)^{\beta} \tag{2.21}$$

Regarding the expenditure function, we may also inspect its optimal behavior from duality manner. Namely solve the expenditure minimum's model in the certain level of the utility. As an example, we still take the utility function of two variables to set up the model as (2.22).

$$\begin{aligned} &\underset{x_1 x_2}{Min}\, p_1x_1 + p_2x_2 \\ &st. \qquad x_1^{\alpha}\, x_2^{\beta} \geq u^* \end{aligned} \tag{2.22}$$

Therefore, when the utility is the C-D function, assume that η>0, its Lagrange form is:

$$L = p_1 x_1 + p_2 x_2 + \eta(u^* - x_1^{\alpha} x_2^{\beta}) \tag{2.23}$$

Then we obtain the first order condition is:

$$\begin{cases} \dfrac{\partial L}{\partial x_1} = p_1 - \eta\alpha \dfrac{x_1^{\alpha} x_2^{\beta}}{x_1} = 0 \\ \dfrac{\partial L}{\partial x_2} = p_2 - \eta\beta \dfrac{x_1^{\alpha} x_2^{\beta}}{x_2} = 0 \\ \dfrac{\partial L}{\partial \eta} = u^* - x_1^{\alpha} x_2^{\beta} = 0 \end{cases} \tag{2.24}$$

When the level of the utility is fixed, through solving those equations, we obtain the demand function of relying on the utility and the price, which is often called the Hicksian demand function.

$$\begin{cases} x_1^h(p_1, p_2, u) = u^{1/(\alpha+\beta)} \left(\dfrac{p_2\alpha}{p_1\beta} \right)^{\beta/(\alpha+\beta)} \\ x_2^h(p_1, p_2, u) = u^{1/(\alpha+\beta)} \left(\dfrac{p_1\beta}{p_2\alpha} \right)^{\alpha/(\alpha+\beta)} \end{cases} \tag{2.25}$$

Substitute (2.25) into the expenditure function $Y(x_1, x_2)$, we can get an indirect expenditure function e(p, u):

$$e(p_1, p_2, u) = u^{\frac{1}{\alpha+\beta}} p_1^{\frac{\alpha}{\alpha+\beta}} p_2^{\frac{\beta}{\alpha+\beta}} \alpha^{\frac{-\alpha}{\alpha+\beta}} \beta^{\frac{-\beta}{\alpha+\beta}} (\alpha+\beta) = y \tag{2.26}$$

Therefore, an indirect utility function can be obtained:

$$u = v(p_1, p_2, y) = \left(\frac{\alpha}{p_1} \right)^{\alpha} \left(\frac{\beta}{p_2} \right)^{\beta} \left(\frac{y}{\alpha+\beta} \right)^{\alpha+\beta} \tag{2.27}$$

Obviously, the results of equations (2.27) and (2.21) are identical, which shown the indirect utility function directly relies on the price and the income, but it is not directly expressed by the function of the goods consumption in quantity of x_1 and x_2. This indicates that whether a consumer obtaining the optimal satisfaction is decided by the influence of market prices and his income, which will include more market uncertainty factors. Therefore, duality analysis can help us to gain market economic problems deeper scientific understanding.

2.3.5 The Expectation Utility and the Mean-Square Deviation

We have researched the relationships of the risk and the utility. When further to study agricultural production and business risk, we cannot neglect the direct influence of the uncertainty in the utility analysis, since the uncertainty as a more directly existent factor would appear. So that, we will make the problem clear by establish the links between the expected utility and the mean square deviation, and solve the problem of agricultural production and management risk composition. Those related concepts and methods, such as the expected utility, the variance and the correlation coefficient of portfolio, should be considered in several of risk management activities, the questions generally have the following expression.

I. The Expected Utility of Multiple Business Activities

Generally, assuming m random variable x_i $(i=1,2,\cdots,m)$ express for a series of business activities, which can bring certain kind of composed revenue. For each random variable represented in the business activities, μ_i is the corresponding expected revenue. Then we may define the average value as expected revenue (or expected utility) of the portfolio in the whole business activities:

$$\mu(x) = (\mu_1, \mu_2, \cdots, \mu_m)\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}$$

II. The Revenue Variance of the Business Portfolio

Similarly, assuming that m random variable x_i are a series of business activities, which bring certain kinds of the portfolio revenue, thus the corresponding risk in this business activities can be expressed by the variance:

$$\sigma^2(x) = (x_1, x_2, \cdots, x_m)\begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1m} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{m1} & \sigma_{m2} & \cdots & \sigma_{mm} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}$$

Among (i, j=1, 2… m), only if i=j, σ_{ij} is the mean-square deviation (or the standard deviation) for a certain kind of the business activities; Then if i≠j, σ_{ij} is the covariance for the two different kinds of the business activities, writes as Cov (x_i, x_j). Therefore, the portfolio risk in the business activities can also be expressed by the following formula.

$$\sigma(x) = \sqrt{\sum_{i=1}^{m} x_i^2 \sigma_i^2 + \sum_{i\neq}^{m}\sum_{j}^{m} x_i x_j Cov(x_i, x_j)} = \sqrt{\sum_{i=1}^{m} x_i^2 \sigma_i^2 + 2\sum_{i=1}^{m-1}\sum_{j=i+1}^{m} x_i x_j Cov(x_i, x_j)}$$

III. The Correlation Coefficient

The concept of the correlation coefficient is also very important to the risk analysis. It expresses the relativity between two random variables x_i and x_j. Generally, we may define correlation coefficient as the following:

$\gamma_{ij} = \dfrac{Cov(x_i, x_j)}{\sigma_i \cdot \sigma_j}$, and generally has, $-1 < \gamma_{ij} < 1$.

The correlation coefficient has the following meanings: (i) if $\gamma_{ij} = -1$, explains that the revenue in these two business activities has the complete negative correlation, namely while one activity raise the revenue, and another activity reduce the revenue in the same proportion, by now the risk in the business portfolio is the minimum, but the revenue is also reducing along with it; (ii) if

γ_{ij}=0, explains that these two business activities are mutually independent in revenue, namely the revenue of one activity doesn't be able to affect the revenue of another activity, when these two business activities are independent, we may obtain certain degree of the risk dispersion; (iii) if γ_{ij}=1, explains that the revenue of these two business activities keep complete positive correlation, namely when the revenue of an activity (i) is rising or declining, the another activity (j) will bring the revenue for the same percentage change in the same direction. By then, the standard deviation of the business activities portfolio is just equal to the weighted average value of the standard deviation of each business activity, means $\sigma = \sum x_i \sigma_i$, namely although the investor possible to obtain a very high revenue, but the risk cannot get any disperser. The latter is the situation of the highest risk, but the risk in the above two kinds of the situations are both be able to dispersed in different degrees.

IV. The Risk Analyzing Can Proof Mean-Square Deviation Identical with Expected Utility

Many models take the mean-square deviation as the criterion of the decision-making in the economical analysis, but they do not directly take the expected utility maximization as the goal. For example, the mathematical programming is in using the approximate method of the expected value and the variance. If there are two random variables, thus the average value and the variance can be expressed generally as following:

$$\mu(a) = a\mu_1 + (1-a)\mu_2$$
$$\sigma^2(a) = a^2\sigma_{11} + 2a(1-a)\sigma_{12} + (1-a)^2\sigma_{22}$$

Therefore, through the value of a, we can make choice at between the expected revenue and the risk (variance). Now considering the correlation coefficient ρ, thus the above variance also may be written as.

$$\sigma^2(a) = a^2\sigma_{11} + 2a(1-a)\sqrt{\sigma_{11}}\sqrt{\sigma_{22}}\rho + (1-a)^2\sigma_{22}$$

The analysis indicates that the mean-square deviation and the expected utility have the same solution, so understanding the result of this analysis is very important on the future application. When the utility function is a quadratic equation, any distribution will create an equation of the mean-square deviation. Because every distribution function can be portrayed by its matrix generating function, that is also called “the moment generating function (MGF) of the random variable x [34].

Assuming, a k-order moment for a random variable x can be defined as:

$$E(x^k) = \int_{-\infty}^{\infty} x^k f(x)dx$$

Then its moment generating function may be defined as [34] [35].

$$\Psi_x(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} dF(x) = \begin{cases} \int_{-\infty}^{\infty} e^{tx} f(x)dx, & \text{for } X \text{ continuous;} \\ \sum_x e^{tx} p(X = x), & \text{for } X \text{ discrete.} \end{cases}$$

So, to the continuous we may prove:

$$\psi_x^n(0) = \frac{d^n}{dt^n}\psi_x(t)\Big|_{t=0} = E(x^n), n = 1,2,\cdots\cdots.$$

Proof: Using Taylor’s expansion of e^{tx}, when $x_0 \to 0$ we get:

$$\psi_x(t) = E(e^{tx}) = E[e^0 + te^{t0}(x-0) + \frac{1}{2}t^2e^{t0}(x-0)^2 + \frac{1}{6}t^3e^{t0}(x-0)^3 + \cdots\cdots]$$
$$= 1 + E(x)t + E(x^2)\frac{t^2}{2} + E(x^3)\frac{t^3}{6} + \cdots$$

Note that, for any n-order moment, we have:

$$\psi_x^n = \frac{d^n}{dt^n}\psi_x(t) = E(x^n) + E(x^{n+1})t + E(x^{n+2})t^2 + \cdots$$

When let t=0, namely get: $\psi_x^n(0) = E(x^n)$

The above equation indicates, if exist the generating function of random variable x, then the generating function of x and the distribution function of x are one to one correspondence.

For example, for the matrix generating function of the normal distribution, easy to prove:

$$\psi_x(t) = \int_{-\infty}^{\infty} e^{tx} \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\} dx = \exp\left\{\mu t + \frac{\sigma^2 t^2}{2}\right\}$$

If the random variable has the utility function of the negative exponent: U (x) = -exp (-ρx), as it obeys the normal distribution, then its value of expected utility is:

$$E[U(x)] = \int_{-\infty}^{\infty} -\exp(-\rho x) \frac{1}{\sigma\sqrt{2\pi}} \exp[-\frac{(x-\mu)^2}{2\sigma^2}] dx$$
$$= -\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2 - \rho x] dx$$

Using the transforming function $Z = (x-\mu)/\sigma$ we can substituting $x = Z\sigma - \mu, dx = \sigma dZ$ into the above equation, then we get:

$$\mathrm{E[U(x)]} = -\frac{1}{\sigma\sqrt{2o}} \int_{-\infty}^{\infty} \sigma \exp(-\frac{1}{2}Z^2 - \rho\sigma Z + \rho\mu) dZ$$
$$= -\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \{\sigma \exp[-\frac{1}{2}(Z+\rho\sigma)^2]\}\{\exp[(\rho\mu + \frac{1}{2}\rho^2\sigma^2)]\} dZ$$
$$= -\exp[\rho(\mu + \frac{1}{2}\rho\sigma^2)] \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp[-\frac{1}{2}(Z+\rho\sigma)^2] dZ.$$

Therefore, the expected utility function should be seen as a matrix generating function, namely $E[U(x)] = -\exp[\rho(\mu + \frac{1}{2}\rho\sigma^2)]$.

As we known, for a negative exponent utility function, that ρ just is the Arrow-Pratt risk aversion coefficient. So that, we can use this function set the model to optimize the expected utility of an investment portfolio. Thus, we can have:

$$\begin{aligned} &\underset{x}{Max}\ E[U(x)] \\ &St. \quad \sum_{1}^{n} x_i = 1 \\ &\qquad\quad x_i \geq 0. \end{aligned}$$

The model is equivalent to set up a mathematical programming by using the mean-square deviation. Such as following:

$$\begin{aligned} &Max \quad C'x \\ &st \qquad x'\Omega x \leq b \\ &\qquad\quad x_i \geq 0. \end{aligned}$$

After the suitable reorganization, the above expression may be changed into the following form:

$$\begin{aligned} &Max \quad C'x - \rho / 2x'\Omega x \\ &st. \qquad x_i \geq 0. \end{aligned}$$

Obviously, the maximized choice of the expected utility has contained in the effective set of the minimum of the mean-square deviation. That was once proposed by H. M. Markowitz (1952, 1959), the principle is using the average value to measure the revenue of certain investment portfolio, and using the variance to measure the risk of the investment portfolio. Therefore, deriving from the above analysis in theorem we get two evaluation criteria: namely the "average value-variance" pros and cons standard and the "average value-variance" substitution standard. According to the former, for any congenial risk or investment portfolio, only when its average value increasing is more superior, but its variance augment becomes worse. Considering the latter, when the congenial risk or the investment portfolio has the greater variance, its risk can be compensated by the increase of the average value. Then, if it has the smaller average value, its flaw can be compensated by reducing the variance.

This also can be explained by “Risk-Revenue indifference curve” in figure 2.3 of this chapter. In recent years, this theory often used for the risk programming, but simultaneously found its flaws and limits [31], therefore, the author suggest and proof that the variance can be replaced by the information entropy (in chapter four), and try to amend the theory of H. M. Markowitz by using the method of the entropy.

Summary

This chapter first established the economic relation between the market of the agricultural product and the input-factors, and through economic circulation studies to explore the depth reasons of risk existing in market decision-making. In fact, whether subjective or objective reasons if exist, an economic body always have to face things of containing uncertainty. When people had recognized the concept of risk, which means they jumped on scientific development has entered a new stage. Thus, the risk becomes a product and generates a risk of market transactions, which may be the charm of the market economy. Moreover, as a traditional agricultural industries and markets, is also an important field of classical economics, agricultural risk management research has become more prominent. So applying the new classical theory, through establishing the dual model of production function, to analysis the manifestation of the product, the input-factors, the cost, the output value, the profit and such important economic elements and their targets in the aspects of the market supply and the demand to seek for the mechanism of the grasping, aversion and controlling the risk.

For example, at first by using the Cobb-Douglas production function, we obtained a series of important analyzing results, such as the natures of homogeneous, continuous, concave, convex and elasticity, etc, especially derived out the direct and indirect demand function, supply function, profit function and as well as the cost function. A useful discovery was the elasticity

of production scale by technological constraint, once it considered as the market supply should request $h < ½$.

Next, this chapter embarks from the comparison and the measure of the risk, makes the thorough discussion for risk utility theory. We studied the axiomatization assumptions as a basis for accomplishing of the related risk measure, evaluation and economic analysis. Regarding this, we not only gave the strict ordering axiom, but also the continuity, concavity, convexity and so on, as well as some related natures of the weak ordering axiom.

Third, we have established the basic economic concept of the risk utility theory. At between risk and uncertainty, by means of utility theory to analyze their inevitable links, such as expected utility, mean square deviation and variance. Through the description and analysis on those number relationships, we have derived the Arrow-Pratt risk aversion coefficient and two useful property theorems. Based on the utility function and the coefficient of absolute risk aversion, we can express different people's subjective attitudes and behaviors to deal with the risks, and utilize the indifference curve to describe risk and revenue portfolio or to discuss their risk aversion characteristics.

Fourth, through the special studies in the portfolio variance, the mean-square deviation and the correlation coefficient, we clarified these economic concepts and the statistical significance in the future agro-risk research. Many models are taking the mean-square deviation as the decision criteria in the economic analysis, instead of directly taking the maximization of the expected utility as the goal. Therefore, this chapter uses Taylor’s expansion about e^{tx}, which has proved that the expectation utility and the variance have the identical solution, and this is also the important conclusion of the Markowitz investment portfolio theory. It will be a basic condition for future study on modeling and solving the risk programming.