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Zweier I - Convergent Sequence Spaces and Their Properties

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Preface

Sequence spaces play an important role in various fields of Real Analysis, Complex Analysis, Functional Analysis and Topology. These are very useful tools in demonstrating abstract concepts through constructing examples and counter examples. The topic “Sequence Spaces” is very broad in its own sense as one can study from various point of views, e.g. Schauder decomposition, α –, β –, and γ – duals, matrix transformations, measures of noncompactness, topological properties and geometric properties. The central theme of the present book is to introduce and study Zweier I-Convergent sequence spaces.

The structure of this text is straightforward. There are six chapters devoted to the various aspects of the theory. Each chapter is divided into sections. The numbers in the square brackets refers to the references listed in the bibliography.

As usual chapter 1, is devoted to the background materials which begins with the notations and conventions and some basic definitions which are needed throughout the work. This chapter concludes with an introduction to the Ideals which also includes some elementary properties.

In chapter 2, we introduce the Zweier I-convergent sequence spaces \mathcal{Z}^I , \mathcal{Z}_0^I and \mathcal{Z}_∞^I . We prove the decomposition theorem and study topological, algebraic properties and inclusion relations of these spaces.

In chapter 3, we introduce the Paranorm Zweier I-convergent sequence spaces $\mathcal{Z}^I(q)$, $\mathcal{Z}_0^I(q)$ and $\mathcal{Z}_\infty^I(q)$ for $q = (q_k)$, a sequence of positive real numbers. We study some topological properties, prove the decomposition theorem and study some inclusion relations on these spaces.

In chapter 4, we introduce the sequence spaces $\mathcal{Z}^I(M)$, $\mathcal{Z}_0^I(M)$ and $\mathcal{Z}_\infty^I(M)$ using the Orlicz function M . We study the algebraic properties and inclusion relations on these spaces.

In chapter 5, we introduce the sequence spaces $\mathcal{Z}^I(f)$, $\mathcal{Z}_0^I(f)$ and $\mathcal{Z}_\infty^I(f)$ for a modulus function f and study some of the topological and algebraic properties on these spaces.

In chapter 6, we introduce the sequence spaces $\mathcal{Z}^I(F)$, $\mathcal{Z}_0^I(F)$ and $\mathcal{Z}_\infty^I(F)$ for a sequence of moduli $F = (f_k)$ and study some of the topological and algebraic properties on these spaces.

In chapter 7, This is a precise chapter which is very special as it is designed only to study some inclusion relations between various zweier sequence spaces studied previously.

In chapter 8, we introduce the sequence spaces ${}_2\mathcal{Z}^I(F)$, ${}_2\mathcal{Z}_0^I(F)$ and ${}_2\mathcal{Z}_\infty^I(F)$ for a sequence of moduli $F = (f_k)$ and study some of the topological and algebraic properties on these spaces.

In chapter 9, we introduce the sequence spaces ${}_2\mathcal{Z}^I(f)$, ${}_2\mathcal{Z}_0^I(f)$ and ${}_2\mathcal{Z}_\infty^I(f)$ for a modulus function f and study some of the topological and algebraic properties on these spaces.

In chapter 10, we introduce the sequence spaces ${}_2\mathcal{Z}^I(M)$, ${}_2\mathcal{Z}_0^I(M)$, ${}_2\mathcal{Z}_\infty^I(M)$ for an Orlicz function M and study some of the topological and algebraic properties on these spaces.

The book ends with a fairly exhaustive bibliography of books and research articles consulted for the work.

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