

Chapter 7

On Certain Class of Zweier I-Convergent Sequence Spaces

7.1 Introduction

Theorem 7.1.1. [68, Theorem 2.1] The sets \mathcal{Z} and \mathcal{Z}_0 are the linear spaces with the co-ordinate wise addition and scalar multiplication which are the BK-spaces with the norm

$$\|x\|_{\mathcal{Z}} = \|x\|_{\mathcal{Z}_0} = \|Z^p x\|_c.$$

Theorem 7.1.2. [68, Theorem 2.2] The sequence spaces \mathcal{Z} and \mathcal{Z}_0 are linearly isomorphic to the spaces c and c_0 respectively, i.e $\mathcal{Z} \cong c$ and $\mathcal{Z}_0 \cong c_0$.

Theorem 1.3. [68, Theorem 2.3] The inclusions $\mathcal{Z}_0 \subset \mathcal{Z}$ strictly hold for $p \neq 1$.

7.2 Main Results

Recently Šalát, Tripathy and Ziman[65-66] introduced the following sequence spaces

$$c_0^I = \{(x_k) \in \omega : \{k \in \mathbb{N} : |x_k| \geq \epsilon\} \in I\},$$

$$c^I = \{(x_k) \in \omega : \{k \in \mathbb{N} : |x_k - L| \geq \epsilon\} \in I, \text{ for some } L \in \mathbb{C}\},$$

$$\ell_\infty^I = \{(x_k) \in \omega : \{k \in \mathbb{N} : |x_k| \geq M\} \in I, \text{ for each fixed } M > 0\}.$$

Analogous to Kostyrko, Šalát and Wilczyński[12], Šalát Tripathy and Ziman[65-66], Khan and Ebadullah[29,31,37,38] introduced the following classes of sequences.

$$\mathcal{Z}_0^I = \{x = (x_k) \in \omega : \{k \in \mathbb{N} : I - \lim Z^p x = 0\} \in I\},$$

“If I feel unhappy, I do mathematics to become happy. If I feel happy, I do mathematics to keep happy.” -Paul Turan

$$\mathcal{Z}^I = \{x = (x_k) \in \omega : \{k \in \mathbb{N} : I - \lim Z^p x = L, \text{ for some } L\} \in I\},$$

$$\mathcal{Z}_\infty^I = \{x = (x_k) \in \omega : \sup_k |Z^p x| < \infty\}.$$

In [27] for $q = (q_k)$ a sequence of positive reals

$$\mathcal{Z}_0^I(q) = \{x = (x_k) \in \omega : \{k \in \mathbb{N} : |Z^p x|^{q_k} \geq \epsilon\} \in I\},$$

$$\mathcal{Z}^I(q) = \{x = (x_k) \in \omega : \{k \in \mathbb{N} : |Z^p x - L|^{q_k} \geq \epsilon\} \in I, \text{ for some } L \in \mathbb{C}\},$$

$$\mathcal{Z}_\infty^I(q) = \{x = (x_k) \in \omega : \sup_k |Z^p x|^{q_k} < \infty\}.$$

In [8] for an Orlicz function M and $Z^p x = x'$

$$\mathcal{Z}_0^I(M) = \{x = (x_k) \in \omega : I - \lim M\left(\frac{|x'_k|}{\rho}\right) = 0 \text{ for some } \rho > 0\},$$

$$\mathcal{Z}^I(M) = \{x = (x_k) \in \omega : I - \lim M\left(\frac{|x'_k - L|}{\rho}\right) = 0 \text{ for some } L \text{ and } \rho > 0\},$$

$$\mathcal{Z}_\infty^I(M) = \{x = (x_k) \in \omega : \sup_k M\left(\frac{|x'_k|}{\rho}\right) < \infty \text{ for some } \rho > 0\}.$$

In [29] for a modulus function f

$$\mathcal{Z}_0^I(f) = \{(x_k) \in \omega : \text{for a given } \varepsilon > 0, \{k \in \mathbb{N} : f(|x'_k|) \geq \varepsilon\} \in I\},$$

$$\mathcal{Z}^I(f) = \{(x_k) \in \omega : \text{there is } L \in \mathbb{C} \text{ such that}$$

$$\text{for } \varepsilon > 0, \{k \in \mathbb{N} : f(|x'_k - L|) \geq \varepsilon\} \in I\},$$

$$\mathcal{Z}_\infty^I(f) = \{(x_k) \in \omega : \{k \in \mathbb{N} : f(|x'_k|) \geq M\} \in I, \text{ for each fixed } M > 0\}.$$

In [34] for a sequence of moduli $F = (f_k)$

$$\mathcal{Z}_0^I(F) = \{(x_k) \in \omega : \{k \in \mathbb{N} : f_k(|x'_k|) \geq \varepsilon\} \in I\},$$

$$\mathcal{Z}^I(F) = \{(x_k) \in \omega : \{k \in \mathbb{N} : f_k(|x'_k - L|) \geq \varepsilon, \text{ for some } L \in \mathbb{C}\} \in I\},$$

$$\mathcal{Z}_\infty^I(F) = \{(x_k) \in \omega : \{k \in \mathbb{N} : f_k(|x'_k|) \geq M, \text{ for each fixed } M > 0\} \in I\}.$$

Here we give the canonical inclusion relations

Result 7.2.1. $c_0^I \subset c^I \subset \ell_\infty^I$. (See[41,57,58]).

Result 7.2.2. $\mathcal{Z}_0^I \subset \mathcal{Z}^I \subset \mathcal{Z}_\infty^I$. (See[12]).

Result 7.2.3. $\mathcal{Z}_0^I(q) \subset \mathcal{Z}^I(q) \subset \mathcal{Z}_\infty^I(q)$. (See[27]).

Result 7.2.4. $\mathcal{Z}_0^I(M) \subset \mathcal{Z}^I(M) \subset \mathcal{Z}_\infty^I(M)$. (See[35]).

Result 7.2.5. $\mathcal{Z}_0^I(f) \subset \mathcal{Z}^I(f) \subset \mathcal{Z}_\infty^I(f)$. (See[29]).

Result 7.2.6. $\mathcal{Z}_0^I(F) \subset \mathcal{Z}^I(F) \subset \mathcal{Z}_\infty^I(F)$.

