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## Convolutional-Based Distributed Coded Cooperation for Relay Channels

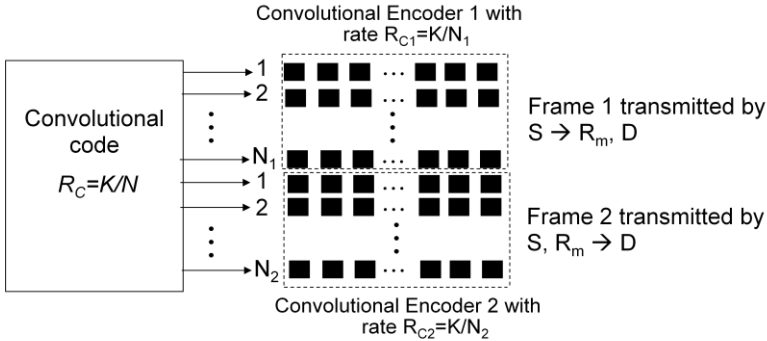


In this chapter, we consider a distributed coded cooperation scheme where the source and relays share their antennas to create a virtual transmit array to transmit towards their destination. While the relays may use several forwarding strategies, including AF and DF, we focus on coded DF relaying. It is assumed that the source is equipped with two encoders, where the output of the first encoder is referred to as the first frame (of length  $N_1$  bits) and the output of the second encoder is referred to as the second frame (of length  $N_2$  bits). Each relay is equipped with an encoder similar to the second encoder at the source. The cooperation scheme under consideration may be summarized as follows. In the first phase, using the first encoder, the source node sends the first frame to the relays and destination node. If a relay successfully decodes the received frame, i.e., the corresponding CRC checks, then the relay encodes the message before transmission. Otherwise, that relay keeps silent. In the second phase, the source and relay nodes (whose CRCs check) transmit the second frame on orthogonal channels (e.g., TDMA, CDMA, or FDMA) to the destination, and the received replicas are combined using maximal-ratio combiner (MRC). The information bits are detected via a Viterbi decoder for the two frames ( $N = N_1 + N_2$  bits). Assuming M-PSK transmission, we analyze the performance of the above distributed coded cooperation scheme and show that it achieves large coding gain and full diversity relative to the coded non-cooperative case. We remark that perfect synchronization is assumed, as is the case in most papers published on this topic.

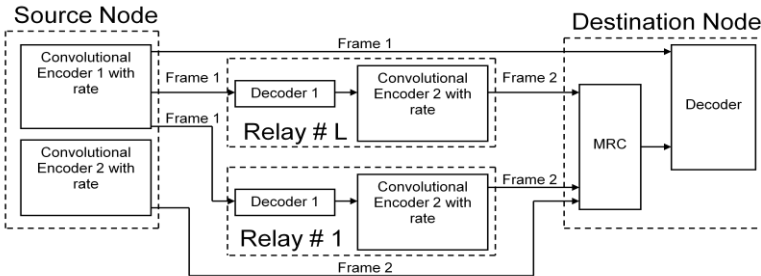
### 3.1 Proposed Coding Scheme

The model of the proposed convolutional encoded transmission system is shown in Figure 3.1. In this model, instead of using a centralized convolutionally coded system at the source node, one can design a distributed coding scheme at

both the source and relay nodes where the encoding process is divided over two frame transmissions. To improve the overall performance through diversity, the coded cooperation operates by sending two code words via  $L+1$  independent fading path, where  $L$  is the number of relay nodes that can be used for cooperation. In what follows, we denote the source,  $m^{\text{th}}$  relay, destination nodes by  $S$ ,  $R_m$ , and  $D$ , respectively. Consider the relay channels shown in Figure 3.2 where data is sent from  $S$  to  $D$  with the assistance of  $R_m$ . All nodes are equipped with single antenna transmitters and receivers. Throughout the book, we assume that a node cannot transmit and receive simultaneously.



**Figure 3.1** Convolutional code designed for distributed coded cooperation.



**Figure 3.2** Proposed distributed coded cooperation scheme.

Let  $b = [b_1, b_2, \dots, b_K]$  be the information sequence at the input of the

convolutional encoder at the source, and let  $C = [c_1, c_2, \dots, c_N]$  be the corresponding code-word. The coded bits are then mapped into a modulated signal word. The coded bits are then mapped into a modulated signal  $x = [x_1, x_2, \dots, x_n]$  the code rate in this case is  $R_c = K/n$  where  $n = \frac{N}{\log_2 M}$ ,  $M$  is the constellation size. According to our coding scheme, codeword  $C$  is partitioned into two sub-code words, namely,  $C_1$  and  $C_2$ , of lengths  $N_1$  and  $N_2$ , respectively, where  $N_1 + N_2 = N$ . Hence, the modulated signal  $x$  is partitioned into two modulated signals, namely,  $x_1$  and  $x_2$ , of lengths  $n_1$  and  $n_2$ , respectively, where  $n_1 + n_2 = n$ .

In the first phase, the source broadcasts the first frame to the relays and destination node using convolutional encoder I with rate  $R_{C_1} = K/n_1$ . If the relays correctly decode the message they received from the source, they re-encode it with convolutional encoder II with rate  $R_{C_2} = K/n_2$ . In the second phase, the source and those relays whose CRC checks transmit the second frame to the destination. The received copies of the second frame are combined using MRC and the information bits are detected via a Viterbi decoder based on the two frames  $N_1 + N_2 = N$ . We assume that all sub-channels are independent, orthogonal, and quasi-static fading. We consider two different convolutional codes, whose generator polynomials in octal form are generally given by  $(c_1, c_2, c_3, c_4)_{\text{octal}}$ . In this context, it is implied that encoder I employs  $(c_1, c_2)_{\text{octal}}$  and encoder II employs  $(c_3, c_4)_{\text{octal}}$ .

During the first frame transmission, the signals received at the relay and the destination nodes at time  $t$  are given by

$$r_{SR_m}(t_1) = \sqrt{R_{C_1} E_{SR_m}} h_{SR_m}(t) x(t) + n_{SR_m}(t), \quad (3.1)$$

$$r_{SD}(t_1) = \sqrt{R_{C1}E_{SD}}h_{SD}(t)x(t) + n_{SD}(t), \quad (3.2)$$

where  $x(t)$  is the output of the source modulator at time slot  $t$  ( $t = 1, 2, \dots, n_1$ ),  $m = 1, 2, \dots, L$ ,  $L$  is the number of relay channels,  $h_{SR_m}(t)$  and  $h_{SD}(t)$  are modeled as complex Gaussian distributed with zero mean and unit variance, representing the fading channels from  $S$  to  $R_m$  and from  $S$  to  $D$ ,  $E_{SR_m}$  and  $E_{SD}$  are the transmitted signal energies for the corresponding link,  $n_{SR_m}(t)$  and  $n_{SD}(t)$  represent the complex AWGN on the  $S - R_m$  and  $S - D$  links, respectively, with zero mean and one-dimensional variance  $N_0/2$ .

Now let  $L'$  denote the number of relays used for cooperation in the second phase, i.e., they decode the received message correctly. Accordingly, the received signals at the destination node at time  $t$  are given by

$$r_{R_mD}(t) = \sqrt{R_{C2} \frac{E_{R_mD}}{(L'+1)}} h_{R_mD}(t) \hat{x}(t) + n_{R_mD}(t), \quad (3.3)$$

$$r_{SD}(t) = \sqrt{R_{C2} \frac{E_{SD}}{(L'+1)}} h_{SD}(t) x(t) + n_{SD}(t), \quad (3.4)$$

where  $\hat{x}(t)$  is the output of the relay modulators at time slot

$t$  ( $t = n_1 + 1, n_1 + 2, \dots, n_1 + n_2$ ),  $m = 1, 2, \dots, L'$ ,  $h_{R_mD}(t)$  is the complex fading coefficient of the  $R_m - D$  link,  $E_{R_mD}$  is the transmitted signal energy for the  $R_m - D$  link,  $n_{R_mD}(t)$  is the AWGN with zero mean and variance  $N_0$ ,  $\frac{1}{(L'+1)}$  is a ratio used to maintain the same average power in the second frame. For example, when  $L' = 0$ , the relay nodes do not transmit. That is, the relay nodes transmit with energy  $\frac{E_{R_mD}}{(L'+1)}$  and the source node transmits with energy  $\frac{E_{SD}}{(L'+1)}$ . Note that the coefficients in (3.3) and (3.4) are functions of  $L'$ , which assumes that power control is used. In the absence of power control,  $L'$  is replaced by  $L$  and consequently the results obtained will serve as upper bounds. However, at

sufficiently high SNR, the difference will be small.

In the following analysis, we consider the performance of our coding scheme over slow fading channels where the fading coefficients remain constant over the transmission of each frame interval, (i.e.,  $h_{SR_m}(t) = h_{SR_m}$ ,  $h_{SD}(t) = h_{SD}$ , and  $h_{R_mD}(t) = h_{R_mD}$ ).

### 3.2 Upper Bound on the Probability of Bit Error

In this section, we evaluate the performance of our proposed scheme for  $L$ -relay channels in terms of the average BER at the destination. In our analysis, we consider  $M$ -PSK modulation. We first consider error-free recovery at the relays. Note that this assumption is optimistic and can only be justified under special conditions (i.e., high SNR or un-faded channel between the source and relays), however, it can serve as a lower bound on the BER performance. Then we consider the effect of channel errors at the relays. Only those relays who correctly decode the message they received from the source node using CRC code, re-encode it with a different code and send it to the destination node, which is more realistic to apply. The instantaneous received SNR for non-cooperative transmission from  $S$  to  $D$ ,

$$\gamma_D(t) = 2R_C \frac{E_{SD}}{N_0} |h_{SD}(t)|^2 = 2R_C \gamma_{SD}(t),$$

and the average SNR

$$\bar{\gamma}_D(t) = 2R_C \frac{E_{SD}}{N_0} E[|h_{SD}(t)|^2] = 2R_C \bar{\gamma}_{SD}(t).$$

The end-to-end conditional pairwise error probability for a coded system is the probability of detecting an erroneous codeword  $\hat{x} = [\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n]$  when in fact  $x = [x_1, x_2, \dots, x_n]$  is transmitted. Therefore, for non-cooperative transmission,

the conditional pairwise error probability from  $S$  to  $D$  is given by

$$P(x \rightarrow \hat{x} | \gamma_{SD}(t)) = Q\left(\sqrt{\frac{2g_{PSK} R_C E_{SD}}{N_0} \sum_{t \in \eta} |h_{SD}(t)|^2}\right) = Q\left(\sqrt{2g_{PSK} R_C \sum_{t \in \eta} \gamma_{SD}(t)}\right), \quad (3.5)$$

where  $g_{PSK} = \sin^2(\pi/M)$ ,  $Q(\cdot)$  is the Gaussian Q-function,  $\eta$  is the set of all  $t$  for which  $\hat{x}(t) \neq x(t)$ , the cardinality of  $\eta$  is equal to the Hamming distance  $d$  between code-words  $x$  and  $\hat{x}$ ,  $\gamma_{SD}(t) = \frac{E_{SD}}{N_0} |h_{SD}(t)|^2$ , and  $R_C$  is the code rate. Under slow fading,  $h_{SD}(t) = h_{SD}$  for all  $t$  and consequently (3.5) can be written as

$$P(d | \gamma_{SD}) = Q(\sqrt{2g_{PSK} R_C d \gamma_{SD}}). \quad (3.6)$$

In what follows, we derive an upper bound on the probability of bit error for the coded  $L$ -relay channels.

### 3.2.1 Distributed Coded Cooperation with Error-Free Relays

Under the assumption of free errors at the relay nodes, the instantaneous received SNR for the channel from  $S$  to  $D$  for the first frame is given by

$$\gamma_D(t) = 2R_{C_1} \frac{E_{SD}}{N_0} |h_{SD}(t)|^2 = 2R_{C_1} \gamma_{SD}(t), \quad t = 1, 2, \dots, n_1, \quad (3.7)$$

and the instantaneous received SNR for the channels from  $S$  to  $D$  and  $R_m$  to  $D$  for the second frame is given by

$$\gamma_D(t) = \frac{2R_{C_1}}{(L+1)} \left( \frac{E_{SD}}{N_0} |h_{SD}(t)|^2 + \sum_{m=1}^L \frac{E_{R_m D}}{N_0} |h_{R_m D}(t)|^2 \right)$$

$$\gamma_D(t) = \frac{2R_{C_1}}{(L+1)} (\gamma_{SD}(t) + \sum_{m=1}^L \gamma_{R_m D}(t)), \quad t = n_1 + 1, n_1 + 2, \dots, n_1 + n_2, \quad (3.8)$$

where  $\gamma_{R_m D}(t) = \frac{E_{R_m D}}{N_0} |h_{R_m D}(t)|^2$ . To maintain the same average power in the



second frame, the relay and source nodes split their powers according to the ratio  $1/(L + 1)$ .

When the fading coefficients  $h_{SD}$ , and  $h_{R_m D}$  are constant over the codeword, the conditional pair-wise error probability is given by

$$P(d|\gamma_{SD}, \gamma_{R_1 D}, \dots, \gamma_{R_L D}) = Q\left(\sqrt{2g_{PSK}\left(R_{C_1}d_1\gamma_{SD} + \frac{R_{C_2}d_2}{(L+1)}(\gamma_{SD} + \sum_{m=1}^L \gamma_{R_m D})\right)}\right), \quad (3.9)$$

where  $\eta_i$  is equal to the Hamming distance  $d_i$  for the two frames,  $i = 1, 2$ , and  $d_1 + d_2 = d$ .

Using Craig's formula for  $Q(x)$  [59]

$$Q(x) = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) d\theta. \quad (3.10)$$

we can rewrite (3.9) as

$$P(d|\gamma_{SD}, \gamma_{R_1 D}, \dots, \gamma_{R_L D}) = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \exp\left(\frac{-g_{PSK}\left(R_{C_1}d_1 + \frac{R_{C_2}d_2}{(L+1)}\right)\gamma_{SD}}{\sin^2\theta}\right) \prod_{m=1}^L \exp\left(\frac{-g_{PSK} R_{C_2} d_2 \gamma_{R_m D}}{(L+1)\sin^2\theta}\right) d\theta. \quad (3.11)$$

The average pairwise error probability is then given by

$$P(d) = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \int_0^\infty \exp\left(\frac{-g_{PSK}\left(R_{C_1}d_1 + \frac{R_{C_2}d_2}{(L+1)}\right)\gamma_{SD}}{\sin^2\theta}\right) p_{\gamma_{SD}}(\gamma_{SD}) d\gamma_{SD} \\ \prod_{m=1}^L \int_0^\infty \exp\left(\frac{-g_{PSK} R_{C_2} d_2 \gamma_{R_m D}}{(L+1)\sin^2\theta}\right) p_{\gamma_{R_m D}}(\gamma_{R_m D}) d\gamma_{R_m D} d\theta, \quad (3.12)$$

where average  $p_\gamma(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$  is a chi-square probability density function (PDF), and  $\bar{\gamma}$  is the average SNR per information bit.

Given the moment generating function (MGF) of  $\gamma$  [60]

$$\psi_\gamma(-s) = \int_0^\infty \exp(-s\gamma) p_\gamma(\gamma) d\gamma = \frac{1}{1+s\bar{\gamma}}. \quad (3.13)$$

One can show that (3.12) can be expressed as

$$P(d) = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \left( 1 + \frac{g_{PSK} \left( R_{C_1} d_1 + \frac{R_{C_2} d_2}{(L+1)} \right) \bar{\gamma}_{SD}}{\sin^2 \theta} \right)^{-1} \\ \cdot \prod_{m=1}^L \left( 1 + \frac{g_{PSK} R_{C_2} d_2 \bar{\gamma}_{R_m D}}{(L+1) \sin^2 \theta} \right)^{-1} d\theta, \quad (3.14)$$

where  $\bar{\gamma}_{SD} = \frac{E_{SD}}{N_0} E[|h_{SD}|^2]$ , and  $\bar{\gamma}_{R_m D} = \frac{E_{R_m D}}{N_0} E[|h_{R_m D}|^2]$  are the average SNRs. If we assume  $\bar{\gamma}_{SD}$ , and  $\bar{\gamma}_{R_m D}$  to be large, then (3.14) can be written as

$$P(d) \approx \left( \sin \frac{\pi}{M} \right)^{-2L-2} \left( \frac{R_{C_2} d_2}{(L+1)} \right)^{-L} \left( R_{C_1} d_1 + \frac{R_{C_2} d_2}{(L+1)} \right)^{-1} (\bar{\gamma}_{SD})^{-1} \\ \cdot \prod_{m=1}^L (\bar{\gamma}_{R_m D})^{-1} \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} (\sin \theta)^{2L+2} d\theta, \quad (3.15)$$

which suggests that the diversity order achieved is  $L+1$  when the channels from  $S$  to the  $L$  relays are error-free. Having obtained the pairwise error probability in (3.15), the BER probability can be upper bounded assuming Gray mapping as [60]

$$P_b(e) \leq \frac{1}{\log_2 M} \frac{1}{k_c} \sum_{d=d_f}^\infty c(d) P(d), \quad (3.16)$$

Where  $k_c$  is the number of information bits encoded into a trellis transition,  $c(d)$  is the sum of bit errors for error events of distance  $d$ , and  $d_f$  is the free Hamming distance of the code.

### 3.2.2 Distributed Coded Cooperation with Errors at Relays

In this section, we consider the more realistic case in which some of the relays may fail to correctly decode the message they received from the source, that is, when their CRC does not check. Obviously, the number of cooperating relays ranges from zero to  $L$ . Let  $\Omega$  denote the set of indices of the cooperating relays, i.e.,

$$\Omega = \{j_1, j_2, \dots, j_{L'}\} \subset \{1, 2, \dots, L\}. \quad (3.17)$$

Note that the cardinality of  $\Omega$  is  $L'$ .

Assuming that  $\gamma_{SD}, \gamma_{SR_1}, \dots, \gamma_{SR_L}, \gamma_{R_1D}, \dots, \gamma_{R_LD}$  are all mutually independent, the expression for the conditional pairwise error probability can be decomposed into three parts. The first part corresponds to the case of no cooperation; the second part corresponds to the case when some of the relays cooperate ( $L'$  of them); and the third part corresponds to the case of error-free relaying, i.e., all relays cooperate. As such, the conditional pairwise error probability can be expressed as

$$\begin{aligned} P(d|\gamma_{SD}, \gamma_{SR_1}, \dots, \gamma_{SR_L}, \gamma_{R_1D}, \dots, \gamma_{R_LD}) &= Q\left(\sqrt{2g_{PSK}(R_{C_1}d_1 + R_{C_2}d_2)\gamma_{SD}}\right) \\ &\quad \cdot \prod_{m=1}^L Q\left(\sqrt{2g_{PSK}R_{C_1}d_1\gamma_{SR_m}}\right) \\ &\quad + \sum_{L'=1}^{L-1} \sum_{\Omega} \left[ \prod_{j \notin \Omega} Q\left(\sqrt{2g_{PSK}R_{C_1}d_1\gamma_{SR_j}}\right) \prod_{j \in \Omega} \left(1 - Q\left(\sqrt{2g_{PSK}R_{C_1}d_1\gamma_{SR_j}}\right)\right) \right. \\ &\quad \cdot \left. Q\left(\sqrt{2g_{PSK}\left(\left(R_{C_1}d_1 + \frac{R_{C_2}d_2}{(L'+1)}\right)\gamma_{SD} + \frac{R_{C_2}d_2}{(L'+1)}\sum_{j \in \Omega} \gamma_{R_jD}\right)}\right) \right] \end{aligned}$$

$$\begin{aligned}
& + \prod_{m=1}^L \left( 1 - Q \left( \sqrt{2g_{PSK} R_{C_1} d_1 \gamma_{SR_m}} \right) \right) \\
& \cdot Q \left( \sqrt{2g_{PSK} \left( \left( R_{C_1} d_1 + \frac{R_{C_2} d_2}{(L+1)} \right) \gamma_{SD} + \frac{R_{C_2} d_2}{(L+1)} \sum_{m=1}^L \gamma_{R_m D} \right)} \right). \quad (3.18)
\end{aligned}$$

Now, using (3.10), (3.18) can then be written with the help of [6] as

$$\begin{aligned}
P(d|\gamma_{SD}, \gamma_{SR_1}, \dots, \gamma_{SR_L}, \gamma_{R_1 D}, \dots, \gamma_{R_L D}) &= \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \\
&\cdot \exp \left( \frac{-g_{PSK} (R_{C_1} d_1 + R_{C_2} d_2) \gamma_{SD}}{\sin^2 \theta} \right) d\theta \prod_{m=1}^L \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \exp \left( \frac{-g_{PSK} R_{C_1} d_1 \gamma_{SR_m}}{\sin^2 \theta_m} \right) \\
&\cdot d\theta_m + \sum_{L=1}^{L-1} \sum_{\Omega} \left[ \prod_{j \notin \Omega} \left( \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \exp \left( \frac{-g_{PSK} R_{C_1} d_1 \gamma_{SR_j}}{\sin^2 \theta_j} \right) d\theta_j \right) \right. \\
&\cdot \prod_{j \in \Omega} \left( 1 - \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \exp \left( \frac{-g_{PSK} R_{C_1} d_1 \gamma_{SR_j}}{\sin^2 \theta_j} \right) d\theta_j \right) \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \\
&\cdot \exp \left( \frac{-g_{PSK} \left( R_{C_1} d_1 + \frac{R_{C_2} d_2}{(L+1)} \right) \gamma_{SD}}{\sin^2 \theta} \right) \exp \left( \frac{-g_{PSK} R_{C_2} d_2}{(L'+1) \sin^2 \theta} \sum_{j \in \Omega} \gamma_{R_j D} \right) d\theta \Big] \\
&+ \prod_{m=1}^L \left( 1 - \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \exp \left( \frac{-g_{PSK} R_{C_1} d_1 \gamma_{SR_m}}{\sin^2 \theta_m} \right) d\theta_m \right) \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \\
&\cdot \exp \left( \frac{-g_{PSK} \left( R_{C_1} d_1 + \frac{R_{C_2} d_2}{(L+1)} \right) \gamma_{SD}}{\sin^2 \theta} \right) \exp \left( \frac{-g_{PSK} R_{C_2} d_2}{(L+1) \sin^2 \theta} \sum_{m=1}^L \gamma_{R_m D} \right) d\theta. \quad (3.19)
\end{aligned}$$

Assuming  $\bar{\gamma}_{SR_m}$ ,  $\bar{\gamma}_{R_mD}$ , and  $\bar{\gamma}_{SD}$  to be large and using the results of Appendix A, the average pairwise error probability can be simplified to

$$\begin{aligned}
 P(d) &\approx \frac{\left(\sin \frac{\pi}{M}\right)^{-2} (\bar{\gamma}_{SD})^{-1}}{(R_{C_1}d_1 + R_{C_2}d_2)} \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \sin^2 \theta d\theta \\
 &\cdot \prod_{m=1}^L \left( \frac{\left(\sin \frac{\pi}{M}\right)^{-2} (\bar{\gamma}_{SR_m})^{-1}}{R_{C_1}d_1} \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \sin^2 \theta_m d\theta_m \right) \\
 &+ \sum_{L'=1}^{L-1} \sum_{\Omega} \left[ \prod_{j \notin \Omega} \left( \frac{\left(\sin \frac{\pi}{M}\right)^{-2} (\bar{\gamma}_{SR_j})^{-1}}{R_{C_1}d_1} \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \sin^2 \theta_j d\theta_j \right) \right. \\
 &\cdot \prod_{j \in \Omega} \left( 1 - \frac{\left(\sin \frac{\pi}{M}\right)^{-2} (\bar{\gamma}_{SR_j})^{-1}}{R_{C_1}d_1} \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \sin^2 \theta_j d\theta_j \right) \frac{\left(\sin \frac{\pi}{M}\right)^{-2} (\bar{\gamma}_{SD})^{-1}}{\left(R_{C_1}d_1 + \frac{R_{C_2}d_2}{(L'+1)}\right)} \\
 &\cdot \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \sin^2 \theta \prod_{j \in \Omega} \left( \frac{\left(\sin \frac{\pi}{M}\right)^{-2} (L'+1) (\bar{\gamma}_{R_jD})^{-1}}{R_{C_2}d_2} \sin^2 \theta \right) d\theta \left. \right] \\
 &+ \prod_{m=1}^L \left( 1 - \frac{\left(\sin \frac{\pi}{M}\right)^{-2} (\bar{\gamma}_{SR_m})^{-1}}{R_{C_1}d_1} \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \sin^2 \theta_m d\theta_m \right) \frac{\left(\sin \frac{\pi}{M}\right)^{-2L-2}}{\left(R_{C_1}d_1 + \frac{R_{C_2}d_2}{(L+1)}\right)} \\
 &\cdot \left( \frac{L+1}{R_{C_2}d_2} \right)^L (\bar{\gamma}_{SD})^{-1} \prod_{m=1}^L (\bar{\gamma}_{R_mD})^{-1} \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} (\sin \theta)^{2L+2} d\theta. \quad (3.20)
 \end{aligned}$$

When  $\bar{\gamma}_{SR_m}$  is very large (i.e.,  $\bar{\gamma}_{SR_m} \rightarrow \infty$ ), all the relay will have perfect detection, and thus (3.20) will be the same as (3.15). This is true since the first two terms of (3.20) go to zeros. However, it should be noted that when  $\bar{\gamma}_{SR_m}$  is

very small, there will be a loss in diversity but the system still offers large coding gains relative to the non-cooperative case. By substituting (3.20) into (3.16), one can obtain an upper bound for the probability of bit error.

### 3.3 Outage Probability Analysis

The outage probability,  $P_{out}$ , is another standard performance criterion for systems operating over slow fading channels [61]. With quasi-static fading, the elements of the sequence  $\{\gamma\}$  of block SNRs are exponentially independent and identically distributed (i.i.d.) with average SNR  $\bar{\gamma}$ . We assume that each codeword is divided into two sub-codewords which may not be of equal length. We denote by  $\beta$  the fraction of time that the source transmits in the first frame and by  $(1 - \beta)$  the fraction of time that the relays and source transmit in the second frame. During the first frame, the source transmits with rate  $R_{C_1} = R_C/\beta$  code, while during the second frame, the source and relays transmit with rate  $R_{C_2} = R_C/(1 - \beta)$  code, where the ratio  $\beta$  ( $0 < \beta < 1$ ). Therefore, in this section, we derive the outage probability of the proposed scheme for  $L$ -relay channels.

First, let us consider non-cooperative direct transmission between the source and destination. During this transmission, the instantaneous capacity  $C(\gamma_{SD}) = \log_2(1 + \gamma_{SD})$  [27]. If a rate  $R_C$  code is used, then the channel will be in outage whenever  $C(\gamma_{SD}) < R_C$ , where  $\{C(\gamma_{SD}) < R_C\}$  is called the outage event. The outage probability is found by integrating the PDF of  $\gamma_{SD}$  over the outage event region, that is

$$P_{out} = Pr\{C(\gamma_{SD}) < R_C\} = Pr\{\gamma_{SD} < 2^{R_C} - 1\} = \int_0^{2^{R_C}-1} p_{\gamma_{SD}}(\gamma_{SD}) d\gamma_{SD}$$

$$= \int_0^{2^{R_C}-1} \frac{1}{\bar{\gamma}_{SD}} \exp\left(\frac{-\gamma_{SD}}{\bar{\gamma}_{SD}}\right) d\gamma_{SD} = 1 - \exp\left(\frac{1-2^{R_C}}{\bar{\gamma}_{SD}}\right). \quad (3.21)$$

In what follows, we derive the outage probability for both error-free and erroneous detection at the relays.

### 3.3.1 Distributed Coded Cooperation with Error-Free Relays

If we consider ideal source-relay links, i.e., no errors at the relays, then the destination node will receive a transmission from source and relays. Thus, outage occurs whenever

$$C(\gamma_{SD}, \gamma_{R_m D}) < R_C, \quad (3.22)$$

Where

$$C(\gamma_{SD}, \gamma_{R_m D}) = \log_2 \left[ (1 + \gamma_{SD})^\beta \left( 1 + \frac{\gamma_{SD}}{(L+1)} + \sum_{m=1}^L \frac{\gamma_{R_m D}}{(L+1)} \right)^{(1-\beta)} \right]. \quad (3.23)$$

The outage probability,  $P_{out}$ , can then be evaluated over the outage region in (3.23) as

$$P_{out} = Pr \left\{ (1 + \gamma_{SD})^\beta \left( 1 + \frac{\gamma_{SD}}{(L+1)} + \sum_{m=1}^L \frac{\gamma_{R_m D}}{(L+1)} \right)^{(1-\beta)} < 2^{R_C} \right\}. \quad (3.24)$$

Since  $\gamma_{SD}, \gamma_{R_1 D}, \gamma_{R_2 D}, \dots, \gamma_{R_L D}$  in (3.24) are always greater than or equal zero, then

$$\gamma_{SD} < (2^{R_C} - 1)(L + 1) \triangleq A_1, \quad (3.25)$$

$$\gamma_{R_m D} < \left( 2^{\left( \frac{R_C}{1-\beta} \right)} - 1 \right) (L + 1) \triangleq A_2, \quad (3.26)$$

where  $m = 1, 2, \dots, L$ .

Now using (3.25) and (3.26) in (3.24), the outage probability is given by

$$\begin{aligned}
P_{out} &= \int_0^{A_1} \frac{1}{\bar{\gamma}_{SD}} \exp\left(\frac{-\gamma_{SD}}{\bar{\gamma}_{SD}}\right) d\gamma_{SD} \\
&\cdot \int_0^{A_2} \cdots \int_0^{A_2} \prod_{m=1}^L \left(\frac{1}{\bar{\gamma}_{R_mD}}\right) \exp\left(-\sum_{m=1}^L \frac{\gamma_{R_mD}}{\bar{\gamma}_{R_mD}}\right) \prod_{m=1}^L d\gamma_{R_mD} \\
&= \left(1 - \exp\left(\frac{-A_1}{\bar{\gamma}_{SD}}\right)\right) \prod_{m=1}^L \left(\int_0^{A_2} \frac{1}{\bar{\gamma}_{R_mD}} \exp\left(\frac{-\gamma_{R_mD}}{\bar{\gamma}_{R_mD}}\right) d\gamma_{R_mD}\right) \\
&= \left(1 - \exp\left(\frac{-A_1}{\bar{\gamma}_{SD}}\right)\right) \prod_{m=1}^L \left(1 - \exp\left(\frac{-A_2}{\bar{\gamma}_{R_mD}}\right)\right). \tag{3.27}
\end{aligned}$$

To show the diversity in (3.27), we consider Taylor series expansion of  $\exp(x)$ . In particular, we consider the first two terms of this expansion. As such,  $P_{out}$ , can be approximated as

$$\begin{aligned}
P_{out} &\approx \frac{A_1}{\bar{\gamma}_{SD}} \prod_{m=1}^L \frac{A_2}{\bar{\gamma}_{R_mD}} \\
&\approx (2^{R_C} - 1)(L + 1) \left[ \left(2^{\left(\frac{R_C}{1-\beta}\right)} - 1\right) (L + 1) \right]^L (\bar{\gamma}_{SD})^{-1} \prod_{m=1}^L (\bar{\gamma}_{R_mD})^{-1}, \tag{3.28}
\end{aligned}$$

which suggests that the diversity order is  $L+1$  when the channel from  $S$  to  $R_m$  is error-free.

### 3.3.2 Distributed Coded Cooperation with Errors at Relays

When a relay is successful in decoding the received message, then that relay will not be in outage, which translates into the following event:  $C(\gamma_{SR_m}) = \beta \log_2(1 + \gamma_{SR_m}) > R_C$ . Otherwise, the relay will be in outage. Thus, we can write the end-to-end outage probability at the destination given the two phases of transmission as



$$\begin{aligned}
 P_{out} = & Pr\{(1 + \gamma_{SD})^\beta (1 + \gamma_{SD})^{(1-\beta)} < 2^{R_C}\} \prod_{m=1}^L Pr\left\{\gamma_{SR_m} < 2^{\frac{R_C}{\beta}} - 1\right\} \\
 & + \sum_{L=1}^{L-1} \sum_{\Omega} \left[ \prod_{j \notin \Omega} Pr\left\{\gamma_{SR_j} < 2^{\frac{R_C}{\beta}} - 1\right\} \prod_{j \in \Omega} Pr\left\{\gamma_{SR_j} > 2^{\frac{R_C}{\beta}} - 1\right\} \right. \\
 & \cdot Pr\left\{(1 + \gamma_{SD})^\beta \left(1 + \frac{1}{(L' + 1)} \left[\gamma_{SD} + \sum_{j \in \Omega} \gamma_{R_j D}\right]\right)^{(1-\beta)} < 2^{R_C}\right\} \\
 & \left. + \prod_{m=1}^L Pr\left\{\gamma_{SR_m} > 2^{\frac{R_C}{\beta}} - 1\right\} \right. \\
 & \cdot Pr\left\{(1 + \gamma_{SD})^\beta \left(1 + \frac{1}{(L+1)} [\gamma_{SD} + \sum_{m=1}^L \gamma_{R_m D}]\right)^{(1-\beta)} < 2^{R_C}\right\}. \quad (3.29)
 \end{aligned}$$

Note that the first term in (3.29) corresponds to the case when all relays are in outage; the second term corresponds to the case when some of the relays are in outage; and the third term corresponds to the case when none of the relays are in outage. It should be clear here that in all three cases the destination is in outage.

The expression in (3.29) can be written in a more compact form as

$$\begin{aligned}
 P_{out} = & I_1 \prod_{m=1}^L \left(1 - \exp\left(\frac{1 - 2^{\frac{R_C}{\beta}}}{\bar{\gamma}_{SR_m}}\right)\right) + \sum_{L=1}^{L-1} \sum_{\Omega} \left[ \prod_{j \notin \Omega} \left(1 - \exp\left(\frac{1 - 2^{\frac{R_C}{\beta}}}{\bar{\gamma}_{SR_j}}\right)\right) \right. \\
 & \cdot \left. \prod_{j \in \Omega} \exp\left(\frac{1 - 2^{\frac{R_C}{\beta}}}{\bar{\gamma}_{SR_j}}\right) I_2(j) \right] + I_3 \prod_{m=1}^L \exp\left(\frac{1 - 2^{\frac{R_C}{\beta}}}{\bar{\gamma}_{SR_m}}\right), \quad (3.30)
 \end{aligned}$$

where  $I_1$ ,  $I_2(j)$ ,  $I_3$ , using Appendix A, are given by

$$\begin{aligned}
 I_1 &= 1 - \exp\left(\frac{(1 - 2^{R_C})}{\bar{\gamma}_{SD}}\right), \quad (3.31) \\
 I_2(j) &= \left(1 - \exp\left(\frac{(1 - 2^{R_C})(L' + 1)}{\bar{\gamma}_{SD}}\right)\right)
 \end{aligned}$$

$$\cdot \prod_{j \in \Omega} \left( 1 - \exp \left( \frac{\left( 1 - 2^{\left( \frac{R_C}{1-\beta} \right)} \right) (L' + 1)}{\bar{\gamma}_{R_j D}} \right) \right), \quad (3.32)$$

$$I_3 = \left( 1 - \exp \left( \frac{(1 - 2^{R_C})(L + 1)}{\bar{\gamma}_{SD}} \right) \right) \cdot \prod_{m=1}^L \left( 1 - \exp \left( \frac{\left( 1 - 2^{\left( \frac{R_C}{1-\beta} \right)} \right) (L+1)}{\bar{\gamma}_{R_m D}} \right) \right). \quad (3.33)$$

Upon applying Taylor series expansion of  $\exp(x)$ , and considering only the first two terms of this expansion, the outage probability in (3.30) is given by

$$\begin{aligned} P_{out} \approx & (2^{R_C} - 1) \left( 2^{\frac{R_C}{\beta}} - 1 \right)^L (\bar{\gamma}_{SD})^{-1} \prod_{m=1}^L (\bar{\gamma}_{SR_m})^{-1} \\ & + \sum_{L=1}^{L-1} \sum_{\Omega} \left[ \prod_{j \notin \Omega} \left( 2^{\frac{R_C}{\beta}} - 1 \right) (\bar{\gamma}_{SR_j})^{-1} \prod_{j \in \Omega} \left( 1 + \frac{\left( 1 - 2^{\frac{R_C}{\beta}} \right)}{\bar{\gamma}_{SR_j}} \right) (2^{R_C} - 1)(L' + 1) \right. \\ & \cdot (\bar{\gamma}_{SD})^{-1} \prod_{j \in \Omega} \left( \left( 2^{\left( \frac{R_C}{1-\beta} \right)} - 1 \right) (L' + 1) (\bar{\gamma}_{R_j D})^{-1} \right) \left. + \prod_{m=1}^L \left( 1 + \frac{\left( 1 - 2^{\frac{R_C}{\beta}} \right)}{\bar{\gamma}_{SR_m}} \right) \right. \\ & \left. (2^{R_C} - 1)(L + 1) \left( \left( 2^{\left( \frac{R_C}{1-\beta} \right)} - 1 \right) (L + 1) \right) (\bar{\gamma}_{SD})^{-1} \prod_{m=1}^L (\bar{\gamma}_{R_m D})^{-1} \right]. \quad (3.34) \end{aligned}$$

Similar to the argument used for (3.20), when  $\bar{\gamma}_{SR_m}$  are very large, (3.34) reduces to (3.28), which proves that the diversity is  $L + 1$  with perfect detection at the relays. When  $\bar{\gamma}_{SR_m}$  are very small, however, the diversity gain diminishes and the system will only offer SNR gains relative to the non-cooperative case.

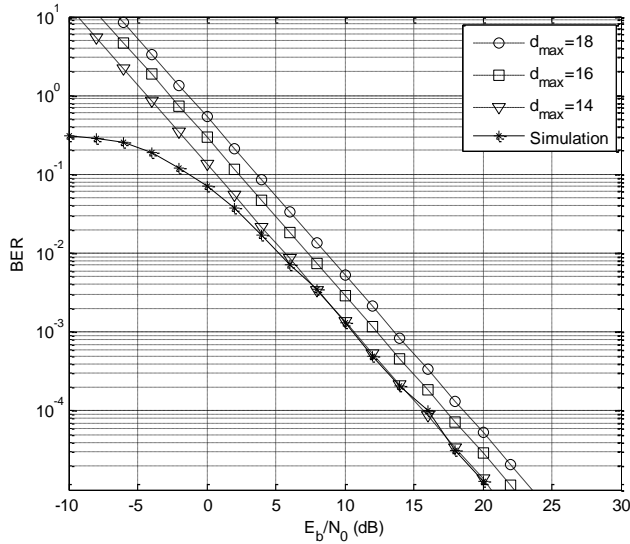
### 3.4 Simulation Results

In our simulations, we assume that the relay nodes operate in the DF mode. For simplicity, BPSK and quadrature phase shift keying (QPSK) modulations are assumed. The different sub-channels between the source, relays and destination are assumed to be independent flat Rayleigh fading channels. Also, we consider a quasi-static fading channel where the channel coefficients are fixed for the duration of the frame and change independently from one frame to another. In all simulations, otherwise mentioned, the transmitted frame size is equal to  $n_1 = n_2 = 130$  coded bits.

The convolutional code used is of constraint length four and generator polynomials  $(13, 15, 15, 17)_{\text{octal}}$  [44]. When the relays cooperate with the source node, the source transmits the codewords corresponding to rate  $1/2$ ,  $(13, 15)_{\text{octal}}$  convolutional code to the relay and destination nodes in the first frame. The relay nodes receive this codeword and decoding is performed to obtain an estimate of the source information bits. In the second frame, the relay and source nodes transmit the codewords on orthogonal channels corresponding to rate  $1/2$ ,  $(15, 17)_{\text{octal}}$  convolutional code to the destination node.

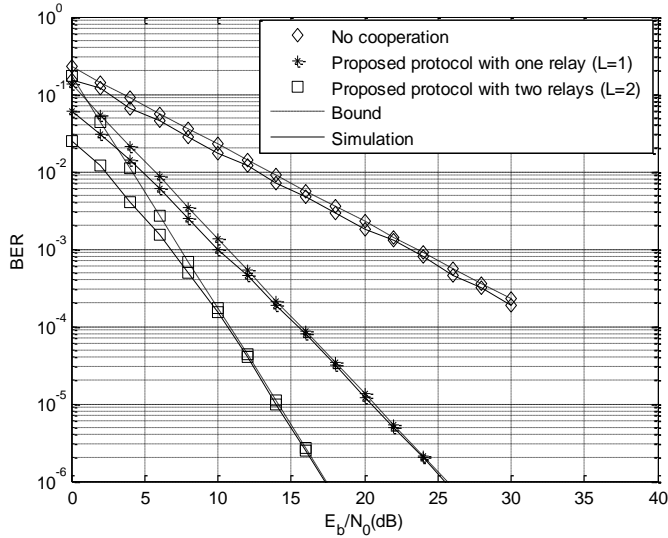
In the analysis, we assume different SNRs between the source and the relay nodes, which is the most general case. This incorporates different network topologies and distances between the source and relay nodes. However, for simplicity, we assume in the simulations that the average SNRs for all sub-channels between the source and relay nodes are equal ( $\bar{\gamma}_{SR_1} = \bar{\gamma}_{SR_2} = \dots = \bar{\gamma}_{SR_L} = \bar{\gamma}_{SR} = \bar{\gamma}_{RD}$ ). Also we assume that the  $R$ - $D$  and  $S$ - $D$  channels have equal SNRs, i.e.,  $\bar{\gamma}_{SD} = \bar{\gamma}_{RD} = E_b/N_0$ , but the  $S$ - $R$  SNR,  $\bar{\gamma}_{SR}$ , can be different.

The union upper bounds on the average bit error probability of the proposed coding scheme operating in the error-free DF mode at relay node for different values of  $d_{\max}$  are shown in Figure 3.3. The code polynomials  $(13, 15, 15, 17)_{\text{octal}}$  and the free distance of this code is  $d_{\text{free}} = 13$ . We also include in the figure, for comparison, the simulated BER results of the proposed coding scheme operating in the error-free DF mode.

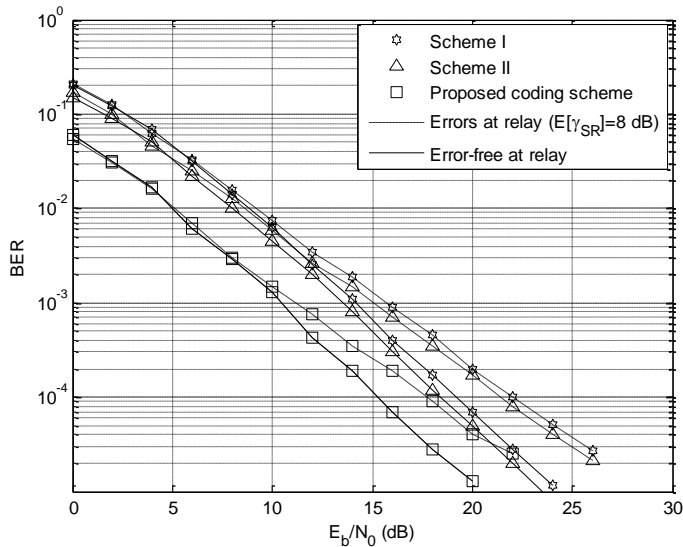


**Figure 3.3** Comparison of analysis and simulated BER with error-free detection at relay node for different values of  $d_{\max}$ ; code  $(13, 15, 15, 17)_{\text{octal}}$  with  $R_{C_1} = R_{C_2} = 0.5$ .

Figure 3.4 shows a comparison of the simulated BER and the analysis in (3.15) and (3.16) for  $L = 1, 2$  relay channels, and  $M = 2$  (BPSK) operating in the error-free DF mode at all relay nodes. In the figure, we include the performance of non-cooperative system as a reference. In non-cooperative case, the source uses convolutional code  $(13, 15, 15, 17)_{\text{octal}}$  of rate  $1/4$ . As shown from these results, for SNR values as high as 10 dB, the analysis is quite tight when compared to simulated results. Also, the diversity gain achieved using different number of relays is evident from these results.



**Figure 3.4** Comparison of the simulated BER and analysis for the proposed coding scheme for  $L = 1, 2$  relay channels, and  $M = 2$  (BPSK) with error-free detection at relay nodes.



**Figure 3.5** The BER performance comparison of proposed coding scheme and the schemes I, II in section 2.2 for  $L = 1$  relay,  $M = 2$  (BPSK) with error-free detection at relay node, and  $\bar{\gamma}_{SR} = 8$  dB with relay errors.

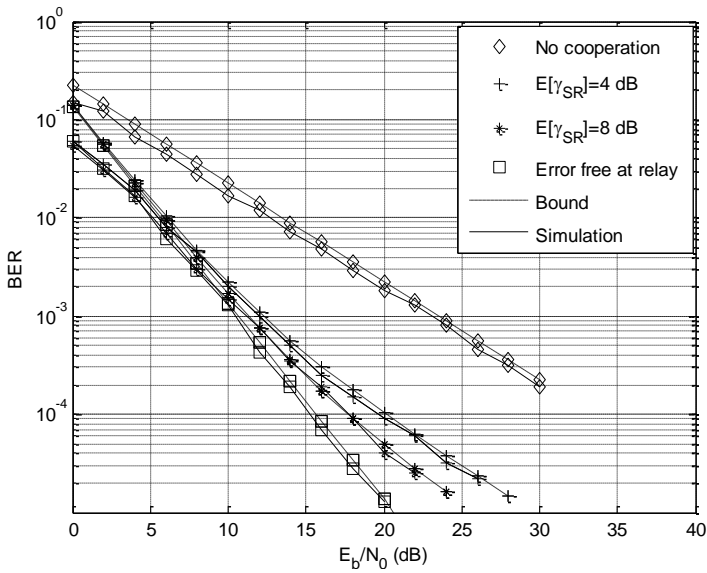
Figure 3.5 shows the BER performance comparison of the proposed coding scheme and the schemes I, II in section 2.2 for both erroneous and error-free detection at the relays. In this figure, we assume  $L=1$  and  $M=2$  (BPSK). To maintain the same average power in the second frame, the source and relay nodes divide their power according to the ratio  $1/(L+1)=1/2$ . As shown in the figure, the performance of the proposed coding scheme is 3 dB better than the scheme II. The performance of the scheme II is 0.5 dB better than the scheme I. The 0.5-dB penalty incurred is due to the use of RCPC code.

Intuitively, when the links between the source and relays are error-free, the system mimics a MIMO system with  $L+1$  transmit antennas where full diversity is always achieved (assuming independent fading channels). On the contrary, when the transmissions from the source to the relays are subject to channel errors, the loss in diversity is mainly a function of  $\bar{\gamma}_{SR}$ . This is clear from Figure 3.6, where we show the same performance plots as a function of  $\bar{\gamma}_{SR}$  with equal transmit power from the relay and source nodes (i.e., in the second frame). The BER curve when the relay is error-free, having diversity order two, is also shown for comparison. The loss of diversity can be clearly observed when the relay is relatively far from the source, resulting in a low  $\bar{\gamma}_{SR}$ .

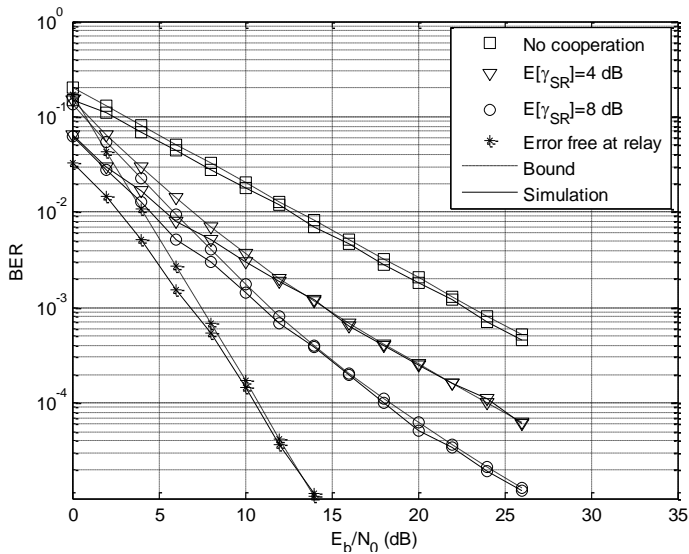
Figure 3.7 shows a comparison between the simulated and the BER upper bound corresponding to (3.16) and (3.20) for  $L=2$  relay channels, and  $M=4$  (QPSK) as a function of  $\bar{\gamma}_{SR}$  with equal transmit power from the relay and source nodes (i.e., in the second frame).

Figure 3.8 shows the outage probability in (3.28) for the proposed transmission scheme (see Figure 3.2) for  $L=1, 2, 3$  relay channels. In this figure, we consider error-free recovery at the relays. As shown, the diversity gain achieved using

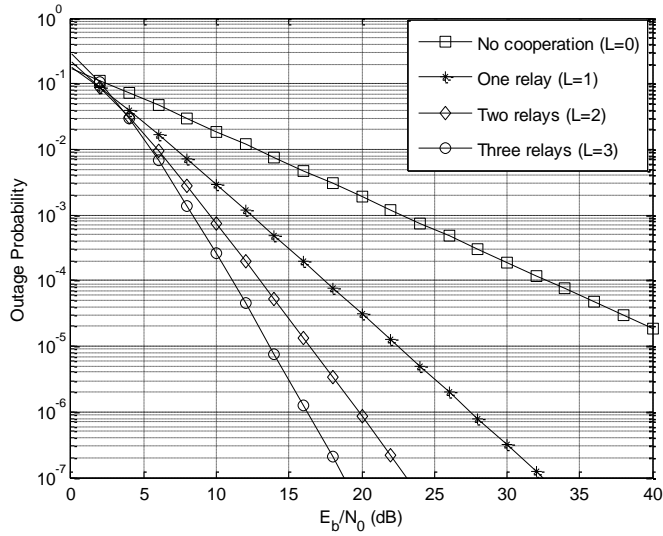
different number of relays is evident from these results.



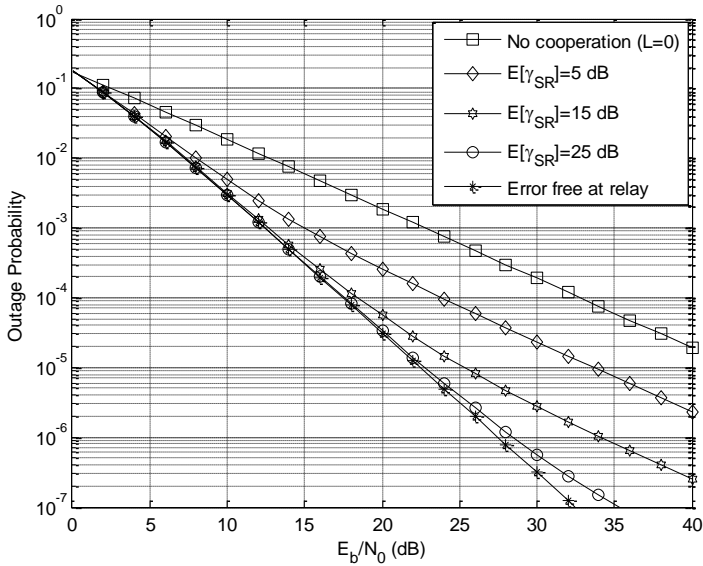
**Figure 3.6** Comparison of analysis and simulated BER for slow Rayleigh fading,  $L=1$  (one relay),  $M=2$  (BPSK), and different  $\bar{\gamma}_{SR}$  with relay errors.



**Figure 3.7** Comparison of analysis and simulated BER for slow Rayleigh fading,  $L=2$  (two relays),  $M=4$  (QPSK), and different  $\bar{\gamma}_{SR}$  with relay errors.



**Figure 3.8** Outage probability for slow Rayleigh fading for  $L = 1, 2$ , and  $3$  relay channels with error-free detection at relay nodes,  $\bar{\gamma}_{SD} = \bar{\gamma}_{RD} = \frac{E_b}{N_0}$ .



**Figure 3.9** Outage probability for slow Rayleigh fading,  $L = 1$  (one relay),  $\beta = 0.5$ , and different  $\bar{\gamma}_{SR}$  with relay errors.

Finally in Figure 3.9, we present the outage probability in (3.34) when the



effect of channel errors at the relay is considered. In this figure, we assume  $L=1$  relay), and  $\beta = 0.5$ . It is clear from this figure that  $\bar{\gamma}_{SR}$  as gets larger, the performance converges to the ideal error-free case. We noted that for  $\bar{\gamma}_{SR} > 25 \text{ dB}$  the diversity order is approximately two. However, for  $\bar{\gamma}_{SR} = 25 \text{ dB}$  this diversity order is slightly reduced. Also when  $\bar{\gamma}_{SR}$  decreases to  $5 \text{ dB}$ , the overall diversity order drops to one.

