5

Coded Cooperative Communications with System Non-idealities

In the previous chapters, we assumed perfect knowledge of the CSI at all network nodes. However, this is an idealistic assumption since such nodes will have to estimate the CSI. The channel estimation technique requires known pilot symbols to be transmitted at the beginning of each data frame. Since pilot symbols reduce spectral efficiency; therefore, it is desirable to use as few as possible. The ultimate objective here in is to achieve performance close to the perfect channel case by using only k pilot symbols.

5.1 Channel Estimation

An important issue affecting the design and analysis of relay transmission protocols is channel state information, i.e., how much radios know about each channel realization throughout the network. For example, using training signals, e.g., pilot tones or symbols, the receivers may estimate the multipath coefficients affecting their respective received signals. Once channel state information is acquired at the distributed radio receivers, protocol designs can feed this information back to the transmitters. This feedback allows the transmitters to adapt their transmissions to the realized channel in effect, often leading to performance improvements when accurate channel state information is obtainable.

5.2 Distributed Coding with MRC Channel Estimation

Recall from Chapters 3 and 4 that we assumed the second frame was transmitted on orthogonal sub-channels (e.g., TDMA, CDMA, or FDMA) from the source and relay nodes to the destination. Also, the detection at the relay and destination nodes was based on perfect channel knowledge. In this chapter, we use the same coding scheme introduced in Chapter 3 with a distributed combining based on Alamouti scheme [9]. For simplicity, we assume that there is one relay. All the nodes are assumed to be equipped with one antenna and one RF chain. Recall that the coding scheme described earlier in Chapter 3, assumes that the source is equipped with two encoders, where the output of the first encoder is referred to as the first frame (of length N_1 bits) and the output of the second encoder is referred to as the second frame (of length N_2 bits). Also, each relay is equipped with an encoder similar to the second encoder at the source. First, at the beginning of each frame transmitted from *S* to *R*, from *R* to *D*, and from *S* to *D*, a pilot sequence *P* consisting of k_p symbols is sent to estimate all the channels. Then, these channel estimates are used to detect the data.

In the following sections, we describe and analyze the pilot-assisted channel estimation technique when employed in the distributed space-time coding scheme in Chapter 3.

5.2.1 System Model

In this section, we introduce the system model of the distributed space-time coding cooperation scheme when employing channel estimation using pilot signals.

Conventional Pilot Mode

In The block diagram with conventional pilot (CP) channel estimation is shown in Figure 5.1. At the beginning of each frame transmitted from *S* to *R*, from *R* to *D*, and from *S* to *D*, a pilot sequence *P* consisting of k_p symbols [45]

$$P = \left(P_1, P_1, \cdots, P_{k_p}\right) \tag{5.1}$$

is appended to the data sequence. A block diagram of the source, relay and destination when employing channel estimation is shown in Figure 5.2.

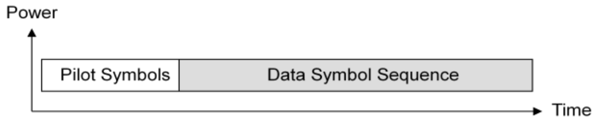

Figure 5.1 *Symbol block with conventional pilot channel estimation.*

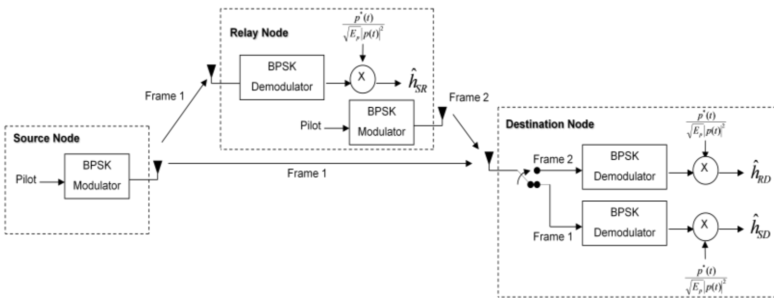

Figure 5.2 *The channel estimation for all the channels for the proposed scheme.*

First Frame: During the first frame transmission, the signals received at the relay and the destination nodes at time t are given respectively by

$$y_{SR}(t) = \sqrt{E_p} h_{SR} p(t) + n_{SR}(t), \tag{5.2}$$

$$y_{SD}(t) = \sqrt{E_p} h_{SD} p(t) + n_{SD}(t), \tag{5.3}$$

where p(t) is the pilot sequence transmitted from the source at time slot t $(t = 1,2,\cdots,k_p)$, E_p is the transmitted signal energy for the pilot sequence, and the rest of the parameters are defined as in Chapter 3.

The receivers at the relay and the destination estimate the channel fading coefficients h_{SR} and h_{SD} by using the observed sequences $y_{SR}(t)$ and $y_{SD}(t)$. The estimates of h_{SR} and h_{SD} are given by [45]

$$\hat{h}_{SR} = \frac{y_{SR}(t)p^*(t)}{\sqrt{E_p}|p(t)|^2} = h_{SR} + \frac{n_{SR}(t)p^*(t)}{\sqrt{E_p}|p(t)|^2} = h_{SR} + \Delta h_{SR}, \tag{5.4}$$

$$\hat{h}_{SD} = \frac{y_{SD}(t)p^*(t)}{\sqrt{E_p}|p(t)|^2} = h_{SD} + \frac{n_{SD}(t)p^*(t)}{\sqrt{E_p}|p(t)|^2} = h_{SD} + \Delta h_{SD}, \tag{5.5}$$

where Δh_{SR} and Δh_{SD} are the estimation errors due to the noise, given by

$$\Delta h_{SR} = \frac{n_{SR}(t)p^*(t)}{\sqrt{E_p}|p(t)|^2}, \tag{5.6}$$

$$\Delta h_{SD} = \frac{n_{SD}(t)p^*(t)}{\sqrt{E_p}|p(t)|^2}. \tag{5.7}$$

Since $n_{SR}(t)$ and $n_{SD}(t)$ are complex AWGN on the *S*–*R* and *S*–*D* links, with zero mean and one-dimensional variance $N_0/2$, the estimation errors Δh_{SR} and Δh_{SD} have a zero mean and one-dimensional variance $N_0/(2k_pE_p)$.

Second Frame: During the second frame transmission, the received signal at the destination node is given by

$$y_{RD}(t) = \sqrt{E_p}h_{RD}p(t) + n_{RD}(t), \tag{5.8}$$

where h_{RD} is modeled as complex Gaussian distributed with zero mean and unit variance, representing the fading channel from *R* to *D*, $n_{RD}(t)$ is the AWGN on the *R*–*D* link with zero mean and one-dimensional variance $N_0/2$.

The receiver at the destination estimates the channel fading coefficient h_{RD} by using the observed sequence $y_{RD}(t)$. The estimate of h_{RD} is then given by

$$\hat{h}_{RD} = \frac{y_{RD}(t)p^*(t)}{\sqrt{E_p}|p(t)|^2} = h_{RD} + \frac{n_{RD}(t)p^*(t)}{\sqrt{E_p}|p(t)|^2} = h_{RD} + \Delta h_{RD}, \tag{5.9}$$

where Δh_{RD} is the estimation error due to the noise, given by

$$\Delta h_{RD} = \frac{n_{RD}(t)p^*(t)}{\sqrt{E_p}|p(t)|^2}. \tag{5.10}$$

Since $n_{RD}(t)$ is complex AWGN, with zero mean and one-dimensional variance $N_0/2$, the estimation error Δh_{RD} has a zero mean and one-dimensional variance $N_0/(2k_pE_p)$.

Data Mode

Having obtained the channel estimates as described earlier, the data mode starts where these estimates are used to detect the transmitted data.

The system model for the proposed system in the first and second frame using Alamouti scheme is depicted in Figure 5.3; 5.4. As shown in the figure, the transmitter is equipped with two RSC encoders, denoted by E_1 and E_2, whose rates are R_{C_1} and R_{C_2}, respectively. The relay is also equipped with E_2. The information sequence b is encoded by E_1, resulting in C_1, which is denoted as *Frame 1*. This frame is broadcasted from the source to the relay and destination nodes. If the relay correctly decodes the message it received from the source, it re-encode it by E_2 and transmitted to the destination as *Frame 2* with rate $R_{C_2} = K/n_2$. At the same time, b is encoded at the source by E_2, resulting in C_2 (denoted as *Frame 2*) which in turn is transmitted from the source to the destination. These two copies (of *Frame 2*) of the source and relay whose CRC check transmit the second frame to the destination. The received copies of the second frame are combined using Alamouti scheme and the information bits are detected via a Viterbi decoder based on the two frames $N = N_1 + N_2$. The combiner output is then augmented with *Frame 1* to form a noisy version of C, which is detected at the destination via a Viterbi decoder.

In what follows we mathematically describe the underlying scheme.

First Frame: During the first frame transmission, the signals received at the relay and the destination nodes at time *t* are given by

$$r_{SR}(t) = \sqrt{R_{C_1} E_{SR}}\, h_{SR}\, x(t) + n_{SR}(t), \tag{5.11}$$

$$r_{SD}(t) = \sqrt{R_{C_1} E_{SD}}\, h_{SD}\, x(t) + n_{SD}(t), \tag{5.12}$$

where $x(t)$ is the output of the source modulator at time slot t ($t = 1,2,\cdots,n_1$), E_{SR} and E_{SD} are the transmitted signal energies for the corresponding link, R_{C_1} is the code rate of convolutional *encoder I*.

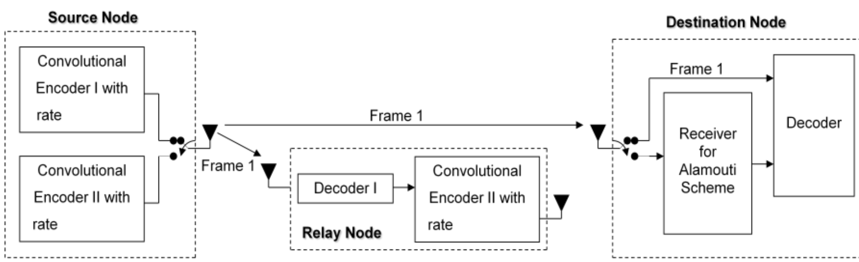

Figure 5.3 *Transmission protocol for the first frame.*

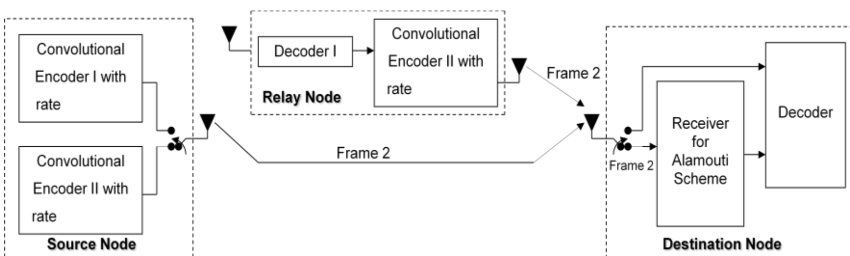

Figure 5.4 *Transmission protocol for the second frame using Alamouti scheme.*

From (5.11) and (5.12), the decision statistics for the channels from S to R and from S to D for the first frame are given respectively, by

$$\tilde{x}_{R,1}(t) = \hat{h}_{SR}^* r_{SR}(t) = (h_{SR}^* + \Delta h_{SR}^*) r_{SR}(t)$$

$$= |h_{SR}|^2 \sqrt{R_{C_1} E_{SR}}\, x(t) + h_{SR}^* n_{SR}(t) + \Delta h_{SR}^* h_{SR} \sqrt{R_{C_1} E_{SR}}\, x(t) + \Delta h_{SR}^* n_{SR}(t), \quad (5.13)$$

$$\tilde{x}_{D,1}(t) = \hat{h}_{SD}^* r_{SD}(t) = (h_{SD}^* + \Delta h_{SD}^*) r_{SD}(t)$$

$$= |h_{SD}|^2 \sqrt{R_{C_1} E_{SD}}\, x(t) + h_{SD}^* n_{SD}(t) + \Delta h_{SD}^* h_{SD} \sqrt{R_{C_1} E_{SD}}\, x(t) + \Delta h_{SD}^* n_{SD}(t), \quad (5.14)$$

where $t = 1,2,\cdots,n_1$. Since the powers of Δh_{SR} and Δh_{SD} in (5.6) and (5.7), respectively, are relatively small (assuming small errors), the powers of $\Delta h_{SR}^* n_{SR}(t)$ and $\Delta h_{SD}^* n_{SD}(t)$ can be neglected.

Second Frame Using Alamouti Scheme: During the second frame transmission, the received signals at the destination node at time *t* and *t* + 1 are given by [9]

$$r_D(t) = \sqrt{R_{C_2}\alpha E_{RD}}h_{RD}\hat{x}(t) + \sqrt{R_{C_2}(1-\alpha)E_{SD}}h_{SD}x(t+1) + n_D(t), \quad (5.15)$$

$$r_D(t+1) = -\sqrt{R_{C_2}\alpha E_{RD}}h_{RD}\hat{x}^*(t) + \sqrt{R_{C_2}(1-\alpha)E_{SD}}h_{SD}x^*(t) + n_D(t+1), \quad (5.16)$$

where $\hat{x}(t)$ and $\hat{x}(t+1)$ are the outputs of the relay modulators at time slot *t* and *t*+1 ($t = n_1 + 1, n_1 + 3, \cdots, n_1 + n_2 - 1$), respectively, the rest of the parameters are defined as before (see Chapter 3).

Using Alamouti's combining scheme [9], from (5.15) and (5.16), the decision statistics for the channels from *R* to *D* and *S* to *D* for the second frame are given respectively by

$$\tilde{x}_{D,2}(t) = \hat{h}_{RD}^* r_D(t) + \hat{h}_{SD} r_D^*(t+1)$$

$$= |h_{RD}|^2\sqrt{R_{C_2}\alpha E_{RD}}\hat{x}(t) + h_{RD}^* h_{SD}\sqrt{R_{C_2}(1-\alpha)E_{SD}}x(t+1) + h_{RD}^* n_D(t)$$

$$-h_{SD}h_{RD}^*\sqrt{R_{C_2}\alpha E_{RD}}\hat{x}(t+1) + |h_{SD}|^2\sqrt{R_{C_2}(1-\alpha)E_{SD}}x(t) + h_{SD}n_D^*(t+1)$$

$$+\Delta h_{RD}^* h_{SD}\sqrt{R_{C_2}(1-\alpha)E_{SD}}x(t+1) + \Delta h_{RD}^* h_{RD}\sqrt{R_{C_2}\alpha E_{RD}}\hat{x}(t)$$

$$+\Delta h_{RD}^* n_D(t) - \Delta h_{SD}h_{RD}^*\sqrt{R_{C_2}\alpha E_{RD}}\hat{x}(t+1)$$

$$+\Delta h_{SD}h_{SD}^*\sqrt{R_{C_2}(1-\alpha)E_{SD}}x(t) + \Delta h_{SD}n_D^*(t+1), \quad (5.17)$$

$$\tilde{x}_{D,2}(t+1) = \hat{h}_{SD}^* r_D(t) - \hat{h}_{RD} r_D^*(t+1)$$

$$= h_{SD}^* h_{RD}\sqrt{R_{C_2}\alpha E_{RD}}\hat{x}(t) + |h_{SD}|^2\sqrt{R_{C_2}(1-\alpha)E_{SD}}x(t+1) + h_{SD}^* n_D(t)$$

$$+|h_{RD}|^2\sqrt{R_{C_2}\alpha E_{RD}}\hat{x}(t+1) - h_{RD}h_{SD}^*\sqrt{R_{C_2}(1-\alpha)E_{SD}}x(t) - h_{RD}n_D^*(t+1)$$

$$+\Delta h_{SD}^* h_{SD}\sqrt{R_{C_2}(1-\alpha)E_{SD}}x(t+1) + \Delta h_{SD}^* n_D(t)$$

$$+\Delta h_{SD}^{*} h_{RD}\sqrt{R_{C_2}\alpha E_{RD}}\hat{x}(t)+\Delta h_{RD} h_{RD}^{*}\sqrt{R_{C_2}\alpha E_{RD}}\hat{x}(t+1)$$

$$-\Delta h_{RD} h_{SD}^{*}\sqrt{R_{C_2}(1-\alpha)E_{SD}}x(t)-\Delta h_{RD} n_D^{*}(t+1), \quad (5.18)$$

where $t = n_1+1, n_1+3, \cdots, n_1+n_2-1$. Since the powers of Δh_{SD} and Δh_{RD} in (5.7) and (5.10), respectively, are relatively small, the powers of $h_{RD}^{*} n_D(t)$, $\Delta h_{SD} n_D^{*}(t+1)$, $\Delta h_{SD}^{*} n_D(t)$, and $\Delta h_{RD} n_D^{*}(t+1)$ can be neglected.

5.2.2 Performance Analysis

In this section, we evaluate the performance of our proposed estimation scheme for one relay channel in terms of the average BER at the destination. In our analysis, we consider M-PSK modulation. We first consider error-free recovery at the relay. Then we consider the effect of channel errors at the relay.

The end-to-end conditional pairwise error probability for a coded system is the probability of detecting an erroneous codeword $\hat{\mathbf{x}} = [\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_n]$, when in fact $\mathrm{x} = [x_1, x_2, \cdots, x_n]$ is transmitted. The instantaneous received SNR for non-cooperative transmission from S to D, $\gamma_D(t) = \frac{2R_C E_{SD}|h_{SD}(t)|^2}{N_0\left(1+\frac{R_C E_{SD}}{k_p E_p}\right)}$. There, for non-cooperative transmission, the conditional pairwise error probability from S to D is given by

$$P(\mathbf{x}\rightarrow\hat{\mathbf{x}}|\gamma_{SD}) = Q\left(\sqrt{\frac{2g_{PSK}R_c E_{SD}}{N_0\left(1+\frac{R_C E_{SD}}{k_p E_p}\right)}\sum_{t\in\eta}|h_{SD}(t)|^2}\right)$$

$$= Q\left(\sqrt{\frac{2g_{PSK}\,R_c}{\left(1+\frac{R_C E_{SD}}{k_p E_p}\right)}\sum_{t\in\eta}\gamma_{SD}(t)}\right). \quad (5.19)$$

Under slow fading, $h_{SD}(t) = h_{SD}$ for all t and consequently (5.19) can be written as

$$P(d|\gamma_{SD}) = Q\left(\sqrt{\frac{2g_{PSK}\,R_c}{\left(1+\frac{R_C E_{SD}}{k_p E_p}\right)}\gamma_{SD}}\right). \tag{5.20}$$

In what follows, we derive an upper bound on the probability of bit error for distributed space-time coding using one relay channel. First, we consider the case of error-free relay then, the case of erroneous relay.

DF with Error-Free Relay

Under the assumption of free errors at the relay node, the instantaneous received SNR for the channel from *S* to *D* for the first frame is given by

$$\gamma_{D_1}(t) = \frac{2R_{C_1}E_{SD}\,|h_{SD}(t)|^2}{N_0\left(1+\frac{R_{C_1}E_{SD}}{k_p E_p}\right)} = \frac{2R_{C_1}\gamma_{SD}(t)}{\left(1+\frac{R_{C_1}E_{SD}}{k_p E_p}\right)}, \quad t = 1,2,\cdots,n_1 \tag{5.21}$$

and the instantaneous received SNR for the channels from *S* to *D* and *R* to *D* for the second frame is given by

$$\gamma_{D_2}(t) = \frac{2R_{C_2}}{N_0}\left(\frac{(1-\alpha)E_{SD}|h_{SD}(t)|^2}{\left(1+\frac{R_{C_2}[(1-\alpha)E_{SD}+\alpha E_{RD}]}{k_p E_p}\right)} + \frac{\alpha E_{RD}|h_{RD}(t)|^2}{\left(1+\frac{R_{C_2}[(1-\alpha)E_{SD}+\alpha E_{RD}]}{k_p E_p}\right)}\right)$$

$$= 2R_{C_2}\left(\frac{(1-\alpha)\gamma_{SD}(t)}{\left(1+\frac{R_{C_2}[(1-\alpha)E_{SD}+\alpha E_{RD}]}{k_p E_p}\right)} + \frac{\alpha\gamma_{RD}(t)}{\left(1+\frac{R_{C_2}[(1-\alpha)E_{SD}+\alpha E_{RD}]}{k_p E_p}\right)}\right)$$

$$t = n_1+1, n_1+2, \cdots, n_1+n_2. \tag{5.22}$$

From (5.21) and (5.22), when the fading coefficients h_{SD}, and h_{RD} are constant over the codeword, the conditional pairwise error probability is given by

$P(d|\gamma_{SD},\gamma_{RD}) =$

$$Q\left(\sqrt{2g_{PSK}\left(\frac{R_{C_1}d_1\gamma_{SD}(t)}{\left(1+\frac{R_{C_1}E_{SD}}{k_pE_p}\right)}+\frac{[R_{C_2}d_2(1-\alpha)E_{SD}+R_{C_2}d_2\alpha E_{RD}]}{1+\frac{R_{C_2}[(1-\alpha)E_{SD}+\alpha E_{RD}]}{k_pE_p}}\right)}\right). \qquad (5.23)$$

Using (3.10), we can rewrite (5.23) as

$$P(d|\gamma_{SD},\gamma_{RD}) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}$$

$$.\exp\left(-g_{PSK}\left(\frac{R_{C_1}d_1}{\left(1+\frac{R_{C_1}E_{SD}}{k_pE_p}\right)}+\frac{R_{C_2}d_2(1-\alpha)}{1+\frac{R_{C_2}[(1-\alpha)E_{SD}+\alpha E_{RD}]}{k_pE_p}}\right)\frac{\gamma_{SD}}{\sin^2\theta}\right)$$

$$.\exp\left(-g_{PSK}\left(\frac{R_{C_2}d_2\alpha}{1+\frac{R_{C_2}[(1-\alpha)E_{SD}+\alpha E_{RD}]}{k_pE_p}}\right)\frac{\gamma_{RD}}{\sin^2\theta}\right)d\theta. \qquad (5.24)$$

The average pairwise error probability is then given by

$$P(d|\gamma_{SD},\gamma_{RD}) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}$$

$$.\int_0^{\infty}\exp\left(-g_{PSK}\left(\frac{R_{C_1}d_1}{\left(1+\frac{R_{C_1}E_{SD}}{k_pE_p}\right)}+\frac{R_{C_2}d_2(1-\alpha)}{1+\frac{R_{C_2}[(1-\alpha)E_{SD}+\alpha E_{RD}]}{k_pE_p}}\right)\frac{\gamma_{SD}}{\sin^2\theta}\right)p_{\gamma_{SD}}(\gamma_{SD})d\gamma_{SD}$$

$$.\int_0^{\infty}\exp\left(-g_{PSK}\left(\frac{R_{C_2}d_2\alpha}{1+\frac{R_{C_2}[(1-\alpha)E_{SD}+\alpha E_{RD}]}{k_pE_p}}\right)\frac{\gamma_{RD}}{\sin^2\theta}\right)p_{\gamma_{RD}}(\gamma_{RD})d\gamma_{RD}d\theta. \qquad (5.25)$$

Using (3.13), one can show that (5.25) can be expressed as

$$P(d) = \frac{1}{\pi} \int_{0}^{\frac{(M-1)\pi}{M}} \left(1 + \frac{g_{PSK}}{\sin^2\theta} \left(\frac{R_{C_2} d_2 \alpha \bar{\gamma}_{RD}}{1 + \frac{R_{C_2}[(1-\alpha)\bar{\gamma}_{SD} + \alpha\bar{\gamma}_{RD}]}{k_p\left(\frac{E_p}{N_0}\right)}} \right) \right)^{-1}$$

$$\cdot \left(1 + \frac{g_{PSK}}{\sin^2\theta} \left(\frac{R_{C_1} d_1 \bar{\gamma}_{SD}}{\left(1 + \frac{R_{C_1}\bar{\gamma}_{SD}}{k_p\left(\frac{E_p}{N_0}\right)}\right)} + \frac{R_{C_2} d_2 (1-\alpha)\bar{\gamma}_{SD}}{1 + \frac{R_{C_2}[(1-\alpha)\bar{\gamma}_{SD} + \alpha\bar{\gamma}_{RD}]}{k_p\left(\frac{E_p}{N_0}\right)}} \right) \right)^{-1} d\theta, \tag{5.26}$$

where $E[|h_{SD}|^2] = 1$, and $E[|h_{RD}|^2] = 1$, are the averages of $|h_{SD}|^2$ and $|h_{RD}|^2$, respectively. Using the results of Appendix C, the average pairwise error probability can be shown as

$$P(d) = \frac{(M-1)}{M} + \frac{A(d)}{(B(d)-A(d))\pi}\sqrt{\frac{A(d)}{1+A(d)}}\tan^{-1}\left(\sqrt{\frac{1+A(d)}{A(d)}}\tan\left(\frac{(M-1)\pi}{M}\right)\right)$$

$$+ \frac{B(d)}{(A(d)-B(d))\pi}\sqrt{\frac{B(d)}{1+B(d)}}\tan^{-1}\left(\sqrt{\frac{1+B(d)}{B(d)}}\tan\left(\frac{(M-1)\pi}{M}\right)\right), \tag{5.27}$$

where

$$A(d) = g_{PSK}\left(\frac{R_{C_1} d_1 \bar{\gamma}_{SD}}{\left(1 + \frac{R_{C_1}\bar{\gamma}_{SD}}{k_p\left(\frac{E_p}{N_0}\right)}\right)} + \frac{R_{C_2} d_2 (1-\alpha)\bar{\gamma}_{SD}}{1 + \frac{R_{C_2}[(1-\alpha)\bar{\gamma}_{SD} + \alpha\bar{\gamma}_{RD}]}{k_p\left(\frac{E_p}{N_0}\right)}} \right), \tag{5.28}$$

and

$$B(d) = g_{PSK}\left(\frac{R_{C_2} d_2 \alpha \bar{\gamma}_{RD}}{1 + \frac{R_{C_2}[(1-\alpha)\bar{\gamma}_{SD} + \alpha\bar{\gamma}_{RD}]}{k_p\left(\frac{E_p}{N_0}\right)}} \right). \tag{5.29}$$

Having obtained the pairwise error probability in (5.27), the BER probability can be upper bounded using (3.16).

Noting that if we assume $\bar{\gamma}_{SD} = \bar{\gamma}_{RD} = E_p/N_0 = E_b/N_0$ to be sufficiently large, the average pairwise error probability can be approximated as

$$P(d) \approx \frac{3(M-1)}{8M}\left(\sin\frac{\pi}{M}\right)^{-4}\left(\frac{R_{C_2}d_2\alpha}{\left(1+\frac{R_{C_2}}{k_p}\right)}\right)^{-1}\left(\frac{R_{C_1}d_1}{\left(1+\frac{R_{C_1}}{k_p}\right)}+\frac{R_{C_2}d_2(1-\alpha)}{\left(1+\frac{R_{C_2}}{k_p}\right)}\right)^{-1}\left(\frac{E_b}{N_0}\right)^{-2}, \quad (5.30)$$

which suggests that the diversity order achieved is two when the channel from S to R is error-free, and the k_p symbols of each pilot sequence or the pilot to noise ratio (PNR), (E_p/N_0) increases.

DF with Errors at Relay

The instantaneous received SNR for the channel from S to R for the first frame is given by

$$\gamma_{D_3}(t) = \frac{2R_{C_1}E_{SR}|h_{SR}(t)|^2}{N_0\left(1+\frac{R_{C_1}E_{SR}}{k_pE_p}\right)} = \frac{2R_{C_1}\gamma_{SR}(t)}{\left(1+\frac{R_{C_1}E_{SR}}{k_pE_p}\right)}, \quad t = 1,2,\cdots,n_1 \quad (5.31)$$

From (5.21), (5.22), and (5.31), when the fading coefficients h_{SD}, h_{RD}, and h_{SR} are constant over the codeword, the conditional pairwise error probability is given by

$$P(d|\gamma_{SD},\gamma_{SR},\gamma_{RD}) = Q\left(\sqrt{2g_{PSK}\left(\frac{R_{C_1}d_1\gamma_{SD}}{\left(1+\frac{R_{C_1}E_{SD}}{k_pE_p}\right)}+\frac{R_{C_2}d_2\gamma_{SD}}{\left(1+\frac{R_{C_2}E_{SD}}{k_pE_p}\right)}\right)}\right)$$

$$.Q\left(\sqrt{2g_{PSK}\frac{R_{C_1}d_1\gamma_{SR}}{\left(1+\frac{R_{C_1}E_{SR}}{k_pE_p}\right)}}\right)+\left(1-Q\left(\sqrt{2g_{PSK}\frac{R_{C_1}d_1\gamma_{SR}}{\left(1+\frac{R_{C_1}E_{SR}}{k_pE_p}\right)}}\right)\right)$$

$$Q\left(\sqrt{2g_{PSK}\left(\frac{R_{C_1}d_1\gamma_{SD}}{\left(1+\frac{R_{C_1}E_{SD}}{k_pE_p}\right)}+\frac{\left[R_{C_2}d_2(1-\alpha)\gamma_{SD}+R_{C_2}d_2\alpha\gamma_{RD}\right]}{\left(1+\frac{R_{C_2}[(1-\alpha)E_{SD}+\alpha E_{RD}]}{k_pE_p}\right)}\right)}\right). \qquad (5.32)$$

Now, using (3.10), (5.32) can be written as

$$P(d|\gamma_{SD},\gamma_{SR},\gamma_{RD}) =$$

$$\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} exp\left(-g_{PSK}\left(\frac{R_{C_1}d_1}{\left(1+\frac{R_{C_1}E_{SD}}{k_pE_p}\right)}+\frac{R_{C_2}d_2}{\left(1+\frac{R_{C_2}E_{SD}}{k_pE_p}\right)}\right)\frac{\gamma_{SD}}{\sin^2\theta_1}\right)d\theta_1$$

$$.\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} exp\left(-g_{PSK}\left(\frac{R_{C_1}d_1}{\left(1+\frac{R_{C_1}E_{SR}}{k_pE_p}\right)}\right)\frac{\gamma_{SR}}{\sin^2\theta_2}\right)d\theta_2$$

$$+\left(1-\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} exp\left(-g_{PSK}\left(\frac{R_{C_1}d_1}{\left(1+\frac{R_{C_1}E_{SR}}{k_pE_p}\right)}\right)\frac{\gamma_{SR}}{\sin^2\theta_1}\right)d\theta_1\right)$$

$$.\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} exp\left(-g_{PSK}\left(\frac{R_{C_1}d_1}{\left(1+\frac{R_{C_1}E_{SD}}{k_pE_p}\right)}+\frac{R_{C_2}d_2(1-\alpha)}{\left(1+\frac{R_{C_2}[(1-\alpha)E_{SD}+\alpha E_{RD}]}{k_pE_p}\right)}\right)\frac{\gamma_{SD}}{\sin^2\theta_2}\right)$$

$$.exp\left(-g_{PSK}\left(\frac{R_{C_2}d_2\alpha}{\left(1+\frac{R_{C_2}[(1-\alpha)E_{SD}+\alpha E_{RD}]}{k_pE_p}\right)}\right)\frac{\gamma_{RD}}{\sin^2\theta_2}\right)d\theta_2. \qquad (5.33)$$

Using (3.13), the average pairwise error probability can then be shown as

$$P(d)=\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\left(1+\frac{g_{PSK}}{\sin^2\theta_1}\left(\frac{R_{C_1}d_1\bar{\gamma}_{SD}}{\left(1+\frac{R_{C_1}\bar{\gamma}_{SD}}{k_p\left(\frac{E_p}{N_0}\right)}\right)}+\frac{R_{C_2}d_2\bar{\gamma}_{SD}}{1+\frac{R_{C_2}\bar{\gamma}_{SD}}{k_p\left(\frac{E_p}{N_0}\right)}}\right)\right)^{-1}d\theta_1$$

$$\cdot\frac{1}{\pi}\int_{0}^{\frac{(M-1)\pi}{M}}\left(1+\frac{g_{PSK}}{\sin^2\theta_2}\left(\frac{R_{C_1}d_1\bar{\gamma}_{SR}}{\left(1+\frac{R_{C_1}\bar{\gamma}_{SR}}{k_p\left(\frac{E_p}{N_0}\right)}\right)}\right)\right)^{-1}d\theta_2$$

$$+\left(1-\frac{1}{\pi}\int_{0}^{\frac{(M-1)\pi}{M}}\left(1+\frac{g_{PSK}}{\sin^2\theta_1}\left(\frac{R_{C_1}d_1\bar{\gamma}_{SR}}{\left(1+\frac{R_{C_1}\bar{\gamma}_{SR}}{k_p\left(\frac{E_p}{N_0}\right)}\right)}\right)\right)^{-1}d\theta_1\right)$$

$$\frac{1}{\pi}\int_{0}^{\frac{(M-1)\pi}{M}}\left(1+\frac{g_{PSK}}{\sin^2\theta_2}\left(\frac{R_{C_1}d_1\bar{\gamma}_{SD}}{\left(1+\frac{R_{C_1}\bar{\gamma}_{SD}}{k_p\left(\frac{E_p}{N_0}\right)}\right)}+\frac{R_{C_2}d_2(1-\alpha)\bar{\gamma}_{SD}}{\left(1+\frac{R_{C_2}[(1-\alpha)\bar{\gamma}_{SD}+\alpha\bar{\gamma}_{RD}]}{k_p\left(\frac{E_p}{N_0}\right)}\right)}\right)\right)^{-1}$$

$$\cdot\left(1+\frac{g_{PSK}}{\sin^2\theta_2}\left(\frac{R_{C_2}d_2\alpha\bar{\gamma}_{RD}}{\left(1+\frac{R_{C_2}[(1-\alpha)\bar{\gamma}_{SD}+\alpha\bar{\gamma}_{RD}]}{k_p\left(\frac{E_p}{N_0}\right)}\right)}\right)\right)^{-1}d\theta_2, \qquad (5.34)$$

where $E[|h_{SR}|^2]=1$ is the average of $|h_{SR}|^2$. Using the results of Appendix C, the average pairwise error probability can be shown as

$$P(d)=\left[\frac{(M-1)}{M}-\frac{1}{\pi}\sqrt{\frac{C(d)}{1+C(d)}}\tan^{-1}\left(\sqrt{\frac{1+C(d)}{C(d)}}\tan\left(\frac{(M-1)\pi}{M}\right)\right)\right]$$

$$\cdot\left[\frac{(M-1)}{M}-\frac{1}{\pi}\sqrt{\frac{D(d)}{1+D(d)}}\tan^{-1}\left(\sqrt{\frac{1+D(d)}{D(d)}}\tan\left(\frac{(M-1)\pi}{M}\right)\right)\right]$$

$$+\left[1-\frac{(M-1)}{M}+\frac{1}{\pi}\sqrt{\frac{D(d)}{1+D(d)}}\tan^{-1}\left(\sqrt{\frac{1+D(d)}{D(d)}}\tan\left(\frac{(M-1)\pi}{M}\right)\right)\right]$$

$$\cdot\left[\frac{(M-1)}{M}+\frac{A(d)}{(B(d)-A(d))\pi}\sqrt{\frac{A(d)}{1+A(d)}}\tan^{-1}\left(\sqrt{\frac{1+A(d)}{A(d)}}\tan\left(\frac{(M-1)\pi}{M}\right)\right)\right.$$

$$\left.+\frac{B(d)}{(A(d)-B(d))\pi}\sqrt{\frac{B(d)}{1+B(d)}}\tan^{-1}\left(\sqrt{\frac{1+B(d)}{B(d)}}\tan\left(\frac{(M-1)\pi}{M}\right)\right)\right], \quad (5.35)$$

where $C(d)=g_{PSK}\left(\frac{R_{C_1}d_1\bar{\gamma}_{SD}}{\left(1+\frac{R_{C_1}\bar{\gamma}_{SD}}{k_p(E_p/N_0)}\right)}+\frac{R_{C_2}d_2\bar{\gamma}_{SD}}{1+\frac{R_{C_2}\bar{\gamma}_{SD}}{k_p(E_p/N_0)}}\right)$, $D(d)=\left(\frac{g_{PSK}\,R_{C_1}d_1\bar{\gamma}_{SR}}{\left(1+\frac{R_{C_1}\bar{\gamma}_{SR}}{k_p(E_p/N_0)}\right)}\right)$,

$A(d)$ and $B(d)$ are defined as in (5.28) and (5.29), respectively. When $\bar{\gamma}_{SR}$ is very large (i.e., $\bar{\gamma}_{SR}\to\infty$), the relay will have perfect detection, and thus (5.35) will be the same as (5.27). Having obtained the pairwise error probability in (5.35), the BER probability can be upper bounded using (3.16).

5.2.3 Simulation Results

In our simulations, we assume that the relay node operates in the DF mode. For simplicity, BPSK modulation is assumed. In all simulations, otherwise mentioned, the transmitted frame size is equal to n1 = n2 = 130 coded bits, and a pilot sequence consisting of kp symbols. The convolutional code used is of constraint length four and generator polynomials $(13, 15, 15, 17)_{\text{octal}}$ [44]. When the relays cooperate with the source node, the source transmits the code-words corresponding to rate 1/2, $(13, 15)_{\text{octal}}$ convolutional code to the relay and destination nodes in the first frame. The relay node receives this codeword and decoding is performed to obtain an estimate of the source information bits. In the second frame, the relay and source nodes transmit the code-words corresponding to rate 1/2, $(15, 17)_{\text{octal}}$ convolutional code using Alamouti scheme to the destination node. Also we assume that the $S-R$, $R-D$, and $S-D$ channels have equal PNRs (E_p/N_0).

Figure 5.5 shows a comparison between the simulated and the bit error rate upper bound corresponding to the expressions given in (5.27), and (3.16) with $k_p = 10$ symbols for three cases of $E_p/N_0 =$ 8, 10, and 14 dB. Code $(13, 15, 15, 17)_{octal}$ is used with $R_{C_1} = R_{C_2} = 0.5$ and α = 0.5. In Figure 5.6, we show a comparison of the simulated and analytical BER results based on the expressions given in (3.16), and (5.35) for $\bar{\gamma}_{SR} = 8$ dB with imperfect channel estimation and $k_p = 10$ symbols for three cases $E_p/N_0 =$ 10, 12, and 16 dB. In addition, we include, for comparison, the results for the DF relaying with errors and perfect channel estimation. It is clear from these figures that as E_p/N_0 gets larger, the performance converges to the ideal case.

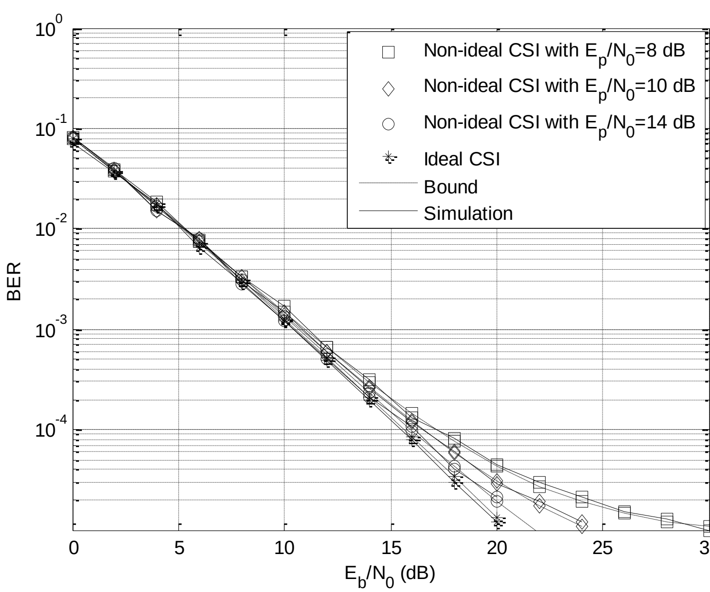

Figure 5.5 *Comparison of analysis and simulated BER with error-free detection at relay node over quasi-static fading;* $k_p = 10$ *symbols;* $E_p/N_0 =$ *8, 10, and 14 dB.*

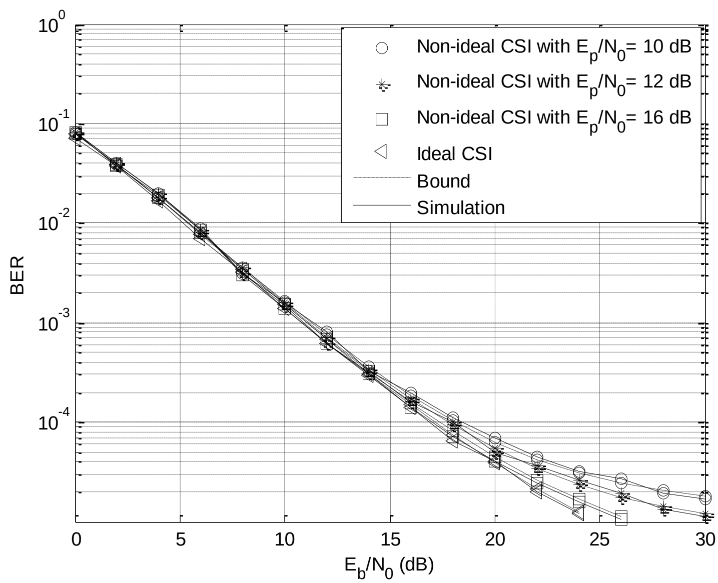

Figure 5.6 *Comparison of analysis and simulated BER for slow Rayleigh fading, $\bar{\gamma}_{SR} = 8$ dB with relay errors, $k_p = 10$ symbols; $E_p/N_0 = 10, 12$, and 16 dB.*