

Mathematical Analysis of Distribution and Redistribution of Income

Johan Fellman



Once I proved a theorem that has
later inspired numerous scientists.

Mathematical Analysis of Distribution and Redistribution of Income

Johan Fellman



SciencePG
Science Publishing Group

Science Publishing Group

548 Fashion Avenue
New York, NY 10018

www.sciencepublishinggroup.com

Published by Science Publishing Group 2015

Copyright © Johan Fellman 2015

All rights reserved.

First Edition

ISBN: 978-1-940366-25-8

This work is licensed under the Creative Commons
Attribution-NonCommercial 3.0 Unported License. To view a copy of
this license, visit

<http://creativecommons.org/licenses/by-nc/3.0/>



or send a letter to:
Creative Commons
171 Second Street, Suite 300
San Francisco, California 94105
USA

To order additional copies of this book, please contact:

Science Publishing Group
book@sciencepublishinggroup.com
www.sciencepublishinggroup.com

Printed and bound in India

Acknowledgements

This work was supported in part

by a grant from the

Magnus Ehrnrooths Stiftelse foundation.

Preface

Different skew models such as the lognormal and the Pareto have been proposed as suitable descriptions of the income distribution, but such specific distributions are usually applied in empirical investigations. For general studies more wide-ranging tools have been considered. The central and most commonly applied theory is connected to the Lorenz curve. Without any assumptions concerning specific distributions, this theory enables analyses of temporal and regional variations in the income inequalities. Particularly, it is a valuable tool for studies of the effect of taxes and transfers to the redistribution of income. Taxation and transferring may have similar effects, but some marked differences with respect to their applications can be identified and therefore, both will usually be given individual presentations.

In this study I have collected the central parts of my contributions to the theory of income distributions and furthermore, I have tried to locate my results within the framework of the general literature.

Contents

Acknowledgements	III
Preface	V
Chapter 1 Introduction	1
1.1 Historical Background.....	3
1.2 Income Distributions	8
1.3 Lorenz Curves and Concentration of Incomes	15
1.4 Modelling Lorenz Curves	26
Chapter 2 Income Transformations	41
2.1 Income Redistributions.....	43
2.2 Additional Properties of Lorenz Curves for Transformed Income Distributions	54
2.3 Regional and Temporal Variation in the Income Inequality	68
2.4 Estimation of Gini Coefficients	73
Chapter 3 Taxation.....	85
3.1 A Class of Tax Policies	87
3.2 Attainable Lorenz Curves	102
3.3 Classes of Non-differentiable Tax Policies	111
3.4 Discussion.....	115
Chapter 4 Transferring.....	117
4.1 The Class of Transfer Policies	119
4.2 Attainable Lorenz Curves	129
4.3 Discontinuous Transfer Policies with a Given Lorenz Curve	136
4.4 Discussion.....	144

Chapter 5 Optimal Redistributive Tax Transfer Policy	149
5.1 Optimal Tax Policy.....	151
5.2 Optimal Transfer Policy	155
5.3 The Optimal Redistributive Tax-transfer Policy	157
5.4 Empirical Illustration: Finland 1971-1990	160
5.5 Concluding Remarks	164