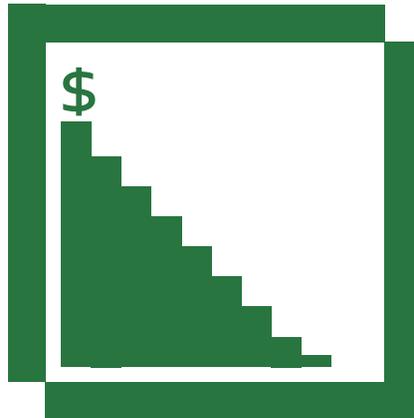


# Mathematical Analysis of Distribution and Redistribution of Income

Johan Fellman



Once I proved a theorem that has  
later inspired numerous scientists.



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$$L_X(p) = \frac{1}{\mu_X} \int_0^{x_p} x f_X(x) dx$$



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## **Preface**

Different skew models such as the lognormal and the Pareto have been proposed as suitable descriptions of the income distribution, but such specific distributions are usually applied in empirical investigations. For general studies more wide-ranging tools have been considered. The central and most commonly applied theory is connected to the Lorenz curve. Without any assumptions concerning specific distributions, this theory enables analyses of temporal and regional variations in the income inequalities. Particularly, it is a valuable tool for studies of the effect of taxes and transfers to the redistribution of income. Taxation and transferring may have similar effects, but some marked differences with respect to their applications can be identified and therefore, both will usually be given individual presentations.

In this study I have collected the central parts of my contributions to the theory of income distributions and furthermore, I have tried to locate my results within the framework of the general literature.



# Contents

Acknowledgements .....	III
Preface .....	V
<b>Chapter 1 Introduction .....</b>	<b>1</b>
1.1 Historical Background.....	3
1.2 Income Distributions .....	8
1.3 Lorenz Curves and Concentration of Incomes .....	15
1.4 Modelling Lorenz Curves .....	26
<b>Chapter 2 Income Transformations .....</b>	<b>41</b>
2.1 Income Redistributions.....	43
2.2 Additional Properties of Lorenz Curves for Transformed Income Distributions .....	54
2.3 Regional and Temporal Variation in the Income Inequality .....	68
2.4 Estimation of Gini Coefficients .....	73
<b>Chapter 3 Taxation.....</b>	<b>85</b>
3.1 A Class of Tax Policies .....	87
3.2 Attainable Lorenz Curves .....	102
3.3 Classes of Non-differentiable Tax Policies .....	111
3.4 Discussion.....	115
<b>Chapter 4 Transferring.....</b>	<b>117</b>
4.1 The Class of Transfer Policies .....	119
4.2 Attainable Lorenz Curves .....	129
4.3 Discontinuous Transfer Policies with a Given Lorenz Curve .....	136
4.4 Discussion.....	144

<b>Chapter 5 Optimal Redistributive Tax Transfer Policy .....</b>	<b>149</b>
5.1 Optimal Tax Policy.....	151
5.2 Optimal Transfer Policy .....	155
5.3 The Optimal Redistributive Tax-transfer Policy .....	157
5.4 Empirical Illustration: Finland 1971-1990 .....	160
5.5 Concluding Remarks .....	164