# Chapter 11

## Preferred and Non-Preferred Reference Frames

## 11.1 Preferred Reference Frame

In this sub-chapter the preferred reference frame $\Sigma'$ is shortly stated. In this frame $\Sigma'$ the metric is the pseudo-Euclidean geometry, i.e.

$$(cd\tau')^2 = (ds')^2 = -\eta_{kl}{}'dx^{k\prime}dx^{l\prime} \tag{11.1a}$$

with

$$\eta_{11}{}' = \eta_{22}{}' = \eta_{33}{}' = 1,\ \eta'_{44} = -1,\ \eta'_{ij} = 0 \text{ (i≠j)}. \tag{11.1b}$$

In addition, the inverse tensor $\eta^{ij\prime}$ is given by

$$\eta'_{ik}\eta^{kj} = \delta_i^j. \tag{11.2a}$$

It follows

$$\eta^{11\prime} = \eta^{22\prime} = \eta^{33\prime} = 1, \eta^{44\prime} = -1, \eta^{ij\prime} = 0 \text{ (i≠j)}. \tag{11.2b}$$

Let $w' = \left(w^1, w^{2\prime}, w^{2\prime}\right)$ be a constant velocity vector and put

$$\gamma = \left(1 - \left|\frac{w\prime}{c}\right|^2\right)^{-1/2}. \tag{11.3}$$

Then, the Lorentz-transformations

$$\begin{aligned} \tilde{x}^{i\prime} &= x^{i\prime} + (\gamma - 1)\frac{(x', w\prime)}{|w\prime|^2}w^{i\prime} + \gamma t' w^{i\prime} \quad \text{(i=1, 2, 3)} \\ c\tilde{t}' &= \gamma\left(ct' + \left(x', \frac{w\prime}{c}\right)\right) \end{aligned} \tag{11.4}$$

do not change the line-element (11.1). All the quantities in $\Sigma'$ are subsequently denoted with a prime and we put

$$x^{4\prime} = ct'. \tag{11.5}$$

The inverse formulae are

$$\begin{aligned} x^{i\prime} &= \tilde{x}^{i\prime} + (\gamma - 1)\frac{(\tilde{x}', w')}{|w'|^2}w^{i\prime} - \gamma\tilde{t}'w^{i\prime} \quad (i = 1, 2, 3) \\ x^{4\prime} &= \gamma\left(\tilde{x}^{4\prime} - \left(\tilde{x}', \frac{w'}{c}\right)\right). \end{aligned} \tag{11.6}$$

Hence, the transformations (11.4) and (11.6) give the possibility to transform a known event in $\Sigma'$ into the same event moving with constant velocity $w'$ in $\Sigma'$.

These are the well-known results of special relativity but the transformations (11.4) and (11.6) are always in in the same frame $\Sigma'$ in contrast to the interpretation of special relativity where the transformations give the same result in a uniformly moving frame with velocity $w'$.The light velocity in the preferred frame $\Sigma'$ is always the vacuum light velocity $c$.

## 11.2 Non-Preferred Reference Frame

Let us now consider a reference frame $\Sigma$ which moves with velocity $-v' = -(v^{1\prime}, v^{2\prime}, v^{3\prime})$ relative to the preferred frame $\Sigma'$. All the results of this sub-chapter can be found in the article [$Pet$ 86].

The non-preferred reference frame $\Sigma$ is received from the preferred frame $\Sigma'$ by the transformations

$$x^i = x^{i\prime} \text{ (i=1,2,3)}, \; x^4 = x^{4\prime} - \left(x', \frac{v\prime}{c}\right). \tag{11.7a}$$

The inverse transformation is

$$x^{i\prime} = x^i, \text{ (i=1,2,3)}, x^{4\prime} = x^4 + \left(x, \frac{v\prime}{c}\right). \tag{11.7b}$$

The metric follows from (11.1). We get

$$\begin{aligned} \eta_{ij} &= \delta_{ij} - \frac{v^{i\prime}}{c}\frac{v^{j\prime}}{c} && \text{(i; j= 1,2,3)} \\ &= -\frac{v^{i\prime}}{c} && \text{(1, 2, 3; j=4)} \\ &= -\frac{v^{j\prime}}{c} && \text{(i=1; j=1, 2, 3)} \\ &= -1. && \text{(i=j=4)} \end{aligned} \tag{11.8a}$$

with

$$(cd\tau)^2 = -\eta_{kl} dx^k dx^l. \tag{11.8b}$$

The inverse has the form

$$\begin{aligned}\eta^{ij} &= \delta^{ij} \quad (i; j = 1,2,3) \\ &= -\frac{v^{i\prime}}{c} \quad (i = 1,2,3;\ j = 4) \\ &= -\frac{v^{j\prime}}{c} \quad (i = 1;\ j = 1,2,3) \\ &= -\left(1 - \left|\frac{v'}{c}\right|^2\right). \quad (i = j = 4)\end{aligned} \tag{11.9}$$

Elementary calculations give the absolute value of light-velocity

$$|v_l| = c/\left(1 - \left|\frac{v'}{c}\right| \cos\vartheta\right) \tag{11.10}$$

where $\vartheta$ denotes the angle between the vectors $v_l$ of light-velocity and $v'$. Hence the light-velocity is anisotropic.

We consider the Michelson-Morley experiment. Let $l$ be the length of the arms of the apparatus. Then, the total time for the travelling of the ray is

$$t = \frac{l}{c}\left\{\left(1 - \left|\frac{v'}{c}\right| \cos\vartheta\right) + \left(1 - \left|\frac{v'}{c}\right| \cos(180^0 - \vartheta)\right)\right\} = \frac{2l}{c}. \tag{11.11}$$

Therefore, the null-result of Michelson-Morley is received. The transformations (11.7) give the result of an event studied in the preferred frame $\Sigma'$ for the same event as it would appear in the non-preferred frame $\Sigma$ and vice versa.

We will now study the transformations in $\Sigma$ which correspond to the Lorentz-transformations in $\Sigma'$, i.e. they transform an event in $\Sigma$ as it appears in $\Sigma$ when it has the velocity $w'$ measured in $\Sigma'$. We have the formulae (11.7) and for the moving object the same transformations hold, i.e.

$$\tilde{x}^{i'} = \tilde{x}^i \quad \text{(i=1,2,3)}, \quad \tilde{x}^{4'} = \tilde{x}^4 + \left(\tilde{x}, \frac{v'}{c}\right). \tag{11.12}$$

The transformations (11.7) and (11.12) yield from the transformations (11.4) by elementary computations the result

$$\tilde{x}^i = x^i + (\gamma - 1)\frac{(x,w\prime)}{|w\prime|^2}w^{i\prime} + \gamma x^4 \frac{w^i\prime}{c} + \gamma\left(x,\frac{v\prime}{c}\right)\frac{w^i\prime}{c}(1,2,3)$$

$$\tilde{x}^4 = \gamma\left(x^4 + \left(x,\frac{w'}{c}\right)\right) - \gamma\left(x^4 + \left(x,\frac{v'}{c}\right)\right)\left(\frac{w'}{c},\frac{v'}{c}\right) + (\gamma-1)\left(\left(x,\frac{v'}{c}\right) - \frac{\left(x,\frac{w'}{c}\right)}{|w'|^2}(w',v')\right). \tag{11.13a}$$

The inverse formulae are

$$x^i = \tilde{x}^i + (\gamma - 1)\frac{(\tilde{x},w\prime)}{|w\prime|^2}w^{i\prime} - \gamma\tilde{x}^4\frac{w^i\prime}{c} - \gamma\left(\tilde{x},\frac{v\prime}{c}\right)\frac{w^i\prime}{c} \ (i=1,2,3)$$

$$x^4 = \gamma\left(\tilde{x}^4 - \left(\tilde{x},\frac{w'}{c}\right)\right) + \gamma\left(\tilde{x}^4 + \left(\tilde{x},\frac{v'}{c}\right)\right)\left(\frac{w'}{c},\frac{v'}{c}\right) + (\gamma-1)\left(\left(\tilde{x},\frac{v'}{c}\right) - \frac{\left(\tilde{x},\frac{w'}{c}\right)}{|w'|^2}(w',v')\right). \tag{11.13b}$$

Any event computed in Σ at rest can be calculated inΣ when it moves with velocity $w'$.

The four-velocity in Σ is

$$\left(\frac{dx^i}{d\tau}\right) = \frac{dt}{d\tau}\left(\frac{dx^1}{dt},\frac{dx^2}{dt},\frac{dx^3}{dt}\right)$$

and in Σ′ the four-velocity is

$$\left(\frac{dx^i\prime}{d\tau\prime}\right) = \frac{dt\prime}{d\tau\prime}\left(\frac{dx^{1\prime}}{dt\prime},\frac{dx^{2\prime}}{dt\prime},\frac{dx^{3\prime}}{dt\prime}\right).$$

The last two relations give by the use of (11.7) and the standard transformations for the velocities in Σ and Σ′:

$$\frac{dx}{dt} = \frac{dx\prime}{dt\prime}\frac{1}{1-\left(\frac{1}{c}\frac{dx\prime}{dt'},\frac{v\prime}{c}\right)} \tag{11.14a}$$

$$\frac{dx'}{dt'} = \frac{dx}{dt}\frac{1}{1+\left(\frac{1}{c}\frac{dx}{dt},\frac{v'}{c}\right)}. \tag{11.14b}$$

In the special case that $\frac{dx^{i'}}{dt'} = v^{i'}$ we get

$$\frac{v}{c} = \frac{v'}{c}\frac{1}{1-\left|\frac{v'}{c}\right|^2}, \quad \frac{v'}{c} = \frac{v}{c}\frac{2}{1+\left(1+4\left|\frac{v}{c}\right|^2\right)^{1/2}}. \tag{11.15}$$

We will now give the transformation formulae for computing in the frame $\Sigma$ an event which takes place in the frame $\Sigma'$. In the frame $\Sigma$ the frame $\Sigma'$ is described by the velocity $w' = v'$ in the formula (11.13), i.e.

$$\begin{aligned} \tilde{x}^i &= x^i + (\gamma - 1)\frac{(x, v')}{|v'|^2}v^{i'} + \gamma x^4 \frac{v^{i'}}{c} + \gamma\left(x, \frac{v'}{c}\right)\frac{v^{i'}}{c} \ (i = 1,2,3) \\ \tilde{x}^4 &= \gamma^{-1}\left(x^4 + \left(x, \frac{v'}{c}\right)\right). \end{aligned} \tag{11.16}$$

The transformation law from $\Sigma$ to $\Sigma'$ is given by (11.7a) which implies

$$\begin{aligned} \tilde{x}^i &= x^{i'} + (\gamma - 1)\frac{(x', v')}{|v'|^2}v^{i'} + \gamma x^{4'} \frac{v^{i'}}{c} \ (i = 1,2,3) \\ \tilde{x}^4 &= \gamma^{-1} x^{4'}. \end{aligned} \tag{11.17a}$$

The inverse formulae are given by

$$\begin{aligned} x^{i'} &= \tilde{x}^i + (\gamma^{-1} - 1)\frac{(\tilde{x}, v')}{|v'|^2}v^{i'} - \gamma \tilde{x}^4 \frac{v^{i'}}{c} \ (i = 1,2,3) \\ x^{4'} &= \gamma \tilde{x}^4. \end{aligned} \tag{1.17b}$$

The formulae (11.17) are given at first by Tangherlini [*Tan* 61] and later on by Marinov [*Mar* 80]. Marinov stated the measurement of the velocity of the Earth of about $\left|\frac{v'}{c}\right| \approx 10^{-3}$ in agreement with the observed velocity relative to the CMB. Hence, we can identify the Earth with the non-preferred frame $\Sigma$ and the CMB frame with the preferred frame $\Sigma'$.

All these results can be found in the article of Petry [*Pet* 86]. Furthermore, the paper contains in the non-preferred frame $\Sigma$ the equations of Maxwell in a medium, the equations of motion of a point particle in the electro-magnetic field.

In addition, the experiments of Hook and Fizeau are studied being in agreement with the observed results. The Doppler-effect is also studied in the reference frame Σ. All these studies are omitted here.